**SECOND TERM: E-LEARNING NOTES**

**SUBJECT: MATHEMATICS CLASS: SS 2**

**SCHEME OF WORK**

**WEEK TOPIC**

**THEME: ALGEBRAIC PROCESSES:**

1. **Revision of last term’s work.**
2. **Logical Reasoning (Revision):**(a) Simple and Compound statements. (b) Logical operation and the truth table. (c) Conditional statements and Indirect proofs.
3. **Linear Inequalities:** (a) Revise linear inequalities in one variable. (b) Solutions of inequalities in two variables. (c) Range of values of combined inequalities.
4. **Linear Inequalities:** (d) Graphs of linear inequalities in two variables. (e) Maximum and minimum values of simultaneous linear inequalities. (f) Application of linear inequalities in real life. (g) Introduction to linear programming.
5. **Algebraic Fractions:** (a) Simplification of fractions. (b) Operation in algebraic fractions. (c) Equation involving fraction

(d)Substitution in fractions. (e) Simultaneous equation involving fractions. (f) Undefined value of a fraction.

**THEME: GEOMETRY:**

1. **Chord property:** (a) Riders based on the circle theorems include: (i) Angles subtended by chords in circle; (ii) Angles subtended by chords at the centre; (iii) Perpendicular bisectors of chords; (iv) Angles in alternate segments.
2. **Circle Theorems:** (a) Proof of: The angle which an arc subtends at the centre is twice the angle it subtends at the circumference. (b) Proof of: Angles in the same segment of a circle are equal. (ii) Angle in a semi-circle. Cyclic quadrilaterals. Tangent to a circle.

**THEME: TRIGONOMETRY:**

1. **Trigonometry:** (a) Derivation of sine rule. (b) Application of sine rule. (c) Derivation and application of cosine rule.
2. **Bearings:** (a) Revision of; Trigonometric ratios; Angles of elevation and depression. (b) Definition and drawing of: (i) 4 cardinal (ii) 8 cardinal points (iii) 16 cardinal points (iv) 32 cardinal points. (c) Notation for bearings: (i) Cardinal notations N300E (ii) S450W (iii) 3-digits notation. E.g. 0750, 3500.
3. **Bearings:** (d) Practical problems on bearing.
4. **Revision.**

**Examination**

**WEEK 1**

**SUBJECT: MATHEMATICS**

**CLASS: SS 2**

**TOPIC: REVISION OF LAST TERM’S WORK**

**TOPIC: Sequence and series**

**Content:**

* Meaning and types of sequences.
* Example of an A.P
* Calculation of (i) First Term (ii) common difference, (iii) nth Term, (iv) Arithmetic mean ,sum of an AP
* Practical problems involving real life situation.

**PERIOD 1**

**SEQUENCES:**

A sequence is an ordered list of numbers whose subsequent values are formed based on a definite rule. The numbers in the sequence are called terms and these terms are normally separated from each other by commas.

#### Examples:

2, 4, 6, 8, 10,……

*Rule: Addition of 2 for subsequent terms*.

70, 66, 62, 58, 54,……

*Rule: Subtraction of 4 for subsequent terms.*

3, -6, 12, -24,……

*Rule: Multiply each term by –2.*

Sequences are either finite or infinite.

A finite sequence is a sequence whose terms can be counted. i.e. it has an end. These types of sequences are usually terminated with a full stop. e.g. (i) 3,5,7,9,11,13. (ii) -7,-10,-13,-16,-19,-21.

If however, the terms in the sequence have no end, the sequence is said to be infinite. These types of sequences are usually ended with three dots, showing that it is continuous. e.g. (i) 5,8,11,14,17,20… (ii) -35,-33,-31,-29,-27,…

**TYPES OF SEQUENCE**

We two types of sequences. They are (i) Arithmetic progression

(ii) Geometric progression

**ARITHMETIC PROGRESSION (AP) {*LINEAR SEQUENCE*}**

If in a sequence of terms T1, T2, T3, ...Tn-1, Tn the difference between any term and the one preceding it is constant, then the sequence is said to be in arithmetic progression (A.P) and the difference is known as the common difference, denoted by d.

∴d = Tn – Tn-1, where n = 1, 2, 3, 4, …

i.e d = T2 – T1 = T3 – T2 = T4 – T3 and so on.

### *Examples of A.P*

(i) 1, 3, 5,7, 9, …

Tn – Tn-1 ⇒ 5 - 3 = 2

7 - 5 = 2

9 - 7 = 2

* d = 2

The difference is common, hence it is an A.P.

(ii) 2, 4, 8, 16, 32, …

Tn – Tn-1 ⇒ 4 - 2 = 2

8 - 4 = 4

16 - 8 = 8

32 - 16 = 16

*The difference is* ***NOT*** *common; therefore it is not an A.P.*

(iii) 70, 66, 62, 58, 54, …

Tn – Tn-1 ⇒ 66 - 70 = -4

62 - 66 = -4

58 - 62 = -4

∴ d = -4

*The difference is common; hence it is an A.P.*

(iv) –2, -5, -8, -11, …

Tn – Tn-1 ⇒ (-5) - (-2) = -5 + 2 = -3

(-8) - (-5) = -8 + 5 = -3

(-11) - (-8) = -11 + 8 = -3.

*The difference is common; hence it is an A.P.*

**EVALUATION**

Which of the following are arithmetic progressing sequence

1. 4,6,8,10,…
2. 3,7,9,11,..
3. 1,6,11,16,21,26…
4. 100,96,92,88,84,…
5. 20,17,15,11,…
6. 45,42,39,36,…

**PERIOD 2**

**THE nth TERM OF AN A.P**

If the first term of an A.P is 3 and the common difference is 2. The terms of the sequence are formed as follows.

1st term = 3

2nd term = 3+2 = 3 + (1)2

3rd term = 3+2+2 = 3 + (2)2

4th term = 3+2+2+2 = 3 + (3)2

5th term = 3+2+2+2+2 = 3 + (4)2

nth term = 3+2+2+2+ … = 3 + (n - 1)2

Hence, the nth term (Tn) of an A.P whose first term is “a” and the common difference is “d” is given as

**Tn = a + (n - 1)d**

#### Example 1:

#### Find the 21st term of the A.P 3, 5, 7, 9, …

#### Solution

a = 3

d = 2

n = 21

Tn = a + (n – 1)d

T21 = 3 + (21 – 1)2

= 3 + 20 x 2

= 3 + 40

= 43.

#### Example 2:

*Find the* 27th *term of the A.P*

100, 96, 92, 88, …

#### Solution

a = 100

d = -4

n = 27

Tn = a + (n – 1)d

T27 = 100 + (27 – 1)(-4)

= 100 + 26 x –4

= 100 – 104

= -4.

## HOW TO SOLVE FOR ‘n’, ‘a’ AND ‘d’

***Example 3:***

*Find the value of n given that* 77 *is the nth term of an A.P* 3½, 7, 10½, …

***Solution:***

a = 3½

d = 7 - 3½

∴ d = 3½

Tn = 77

Tn = a + (n – 1)d

77 = 3½ + (n – 1)3½

77 = 3½ + 3½n – 3½

77 = 3½n

77 = 7/2n

7n = 77 x 2

n = 77 x 2

7

n = 11 x 2

∴n = 22.

***Example 4:***

The question in ***example 1***can be reframed as follows to find a.

“*What is the first term of an A.P whose 21st term is* 43 and *the common difference is* 2 *?”*

***Solution:***

T21 = 43

n = 21

d = 2

Tn = a + (n –1)d

43 = a + 20 x 2

43 = a + 40

a = 43 – 40

∴a = 3

#### Example 5:

The same example in 4 above can be reframed also as follows to find d.

“*Find the common difference of an A.P given that* 43 *is the* 21st *term of the sequence and the first term is* 3*”.*

***Solution:***

a = 3

T21 = 43

n = 21

Tn = a + (n – 1)d

43 = 3 + (21 – 1) d

43 = 3 + 20d

43 –3 = 20d

20d = 40

d = 40

20

∴ d = 2.

**EVALUATION:**

1. Find the 31st term of the sequence –7, -10, -13, -16, …

(2) What is the 26th term of the A.P 5, 10, 15, 20, …?

1. Find the 20th term of the sequence 27, 24, 21, 18, …
2. Find the 18th term of the A. P 6, 12, 18, 24, …
3. Find the 26th term of the A.P –16, -13, -10, -7, …
4. Find the 29th term of the A.P 43, 39, 35, 31, …
5. Find n given that 697 is the nth term of the A.P –3, 4, 11, 18, …
6. Find n given that –8 is the nth term of the A.P 82, 79, 76, 73, 70, …
7. Find n given that 63 is the nth term of the sequence –17, -13, -9, -5, …
8. Find the number of terms in an A.P given that 147 is the last term of the A.P whose first term is 6 and common difference is 3.
9. Find the first term of an A.P given that 124 is the 41st term of the A.P and 3 is the common difference.
10. Find the first term of an A.P given that –5 is the 26th term of the sequence and –3 is the common difference.
11. Given that 57 is the 27th term of an A.P whose common difference is 2, find the first term.
12. Given that 76 is the 21st term of an A.P whose first term is 16. Find the common difference.
13. Given that 67 is the 21st term of an A.P whose first term is 7, find the common difference.
14. Find the common difference of an A.P, given that the 27th term is 90 and the first term is –14.

**PERIOD 3**

#### Example 6:

*The first three terms of an A.P are*

x, 3x + 1*, and* (7x - 4)*. Find the*

*(i) Value of x*

*(ii)* 10th *term*

***Solution:***

(i) Recall that given an A.P T1, T2, T3

T2 – T1 = T3 – T2

Hence for, x, (3x + 1), (7x - 4)

(3x + 1) – x = (7x - 4) – (3x + 1)

3x+1 – x = 7x – 4 – 3x - 1

2x + 1 = 4x – 5

1+ 5 = 4x – 2x

2x = 6

x = 6

2

∴ x = 3.

###### ALTERNATIVE METHOD

a = x

n = 3

d = (3x + 1) – x

= 2x + 1

T3 = 7x – 4

Tn = a + (n - 1)d

7x – 4 = x + (3 - 1)(2x + 1)

7x – 4 = x + 2(2x + 1)

7x – 4 = x + 4x + 2

7x – 5x = 2 + 4

2x = 6

x = 6/2

∴x = 3

The sequence x, (3x + 1), (7x - 4) is

= 3, (3x3 + 1), (7x3 - 4)

= 3, 10, 17.

(ii) a = 3

n = 10

d = 7

Tn = a + (n – 1)d

T10 = 3 + (10 – 1)7

= 3 + 9x7

= 3 + 63

= 66.

***Example 7:***

*The 6th term of an A.P is* –10 *and the 9th term is* –28*.*

*Find the (i) Common difference*

*(ii) First term*

*(iii)* 26th *term of the sequence.*

***Solution:***

(i) T6 = -10 Tn = a + (n - 1)d

n = 6 -10 = a + (6 - 1)d

-10 = a + 5d ----------- (1)

T9 = -28 -28 = a + (9 - 1)d

n = 9 -28 = a + 8d ---------- (2)

Solve equation (1) and (2) simultaneously.

Eqn. (1): -10 = a + 5d

Eqn. (2): -28 = a + 8d

18 = -3d

d = 18

-3

∴ d = -6.

(ii) Put d = -6 in equation (1)

-10 = a + 5(-6)

-10 = a - 30

-10+30 = a

∴ a = 20.

(iii) To find the 26th term of the sequence.

a = 20

d = -6

n = 26

Tn = a + (n - 1)d

T26 = 20 + (26 - 1)(-6)

= 20 + 25(-6)

= 20 - 150

= -130.

## EVALUATION

1. The 6th term of an AP is –10 and the 9th term is 18 less than the 6th term. Find the

(a)common difference (b) first term

(c) 26th term of the sequence.

(2) The 7th term of an AP is 17 and the 13th term is 12 more than the 7th term. Find the (i)           common difference (ii) first term (iii) 21st term of the AP.

(3) The 6th term of an A.P is 26 and the 11th term is 46. Find the

(i) Common difference

(ii) First term

(iii) 25th term of the A.P

(4) The 5th term of an A.P is 11 and the 9th term is 19. Find the (i) common difference

(ii) First term

(iii) 21st term of the A.P

(5) The fourth term of an A.P is 37 and the 6th term is 12 more than the fourth term. Find the first and seventh terms.

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(6) The first three terms of an A.P are (x+2) ,(2x-5) and (4x+1). Find the

(i) Value of x

(ii) 7th term.

(7) The first three terms of an A.P are x, (2x-5) and (x+6). Find the

(i) Value of x

(ii) 21st term.

(8) If the first three terms of an A.P are (4x+1), (2x-5) and (x+3). Find the

(i) Value of x

(ii) Sequence

(iii) 11th term of the sequence

(9) Given that 9, x, y, 24 are in A.P, find the values of x and y.

(10) If –5, a, b, 16 are in A.P, find the values of a and b.

**PERIOD 4**

## Arithmetic Series:

These are series formed from an arithmetic progression. e.g.

1 + 4 + 7 + 10 + …

In general, if Sn is the sum of n terms of an arithmetic series then

Sn = a + (a + d) + (a + 2d) + … + (l - d) +

l -- (1)

Where l is the nth term, a is the first term and d is the common difference.

Rewriting the series above starting with the nth term, we have.

Sn = l +(l - d) + (l - 2d) +… + (a + d) + a ----- (2)

Adding equation (1) and (2) we have

2Sn = (a + l) + (a + l) + … + (a + l) + (a + l) in n places

2Sn = n(a + l)

∴Sn = n/2(a + l)

But l is the nth term i.e a + (n - 1) d

Sn = n/2{a +a + (n - 1)d}

∴ Sn = n/2 {2a + (n - 1)d}

***Example 8 :***

*Find the sum of the first* 20 *terms of the series* 3+5+7+9+ …

***Solution:***

a = 3

d = 2

n = 20

Sn = n/2 {2a +(n - 1)d}

S20 = 20/2 {2x3 +(20 - 1)2}

S20 = 10{6 + 19 x 2}

= 10{6 + 38}

= 10{44}

∴ S20 = 440

***Example 9:***

*Find the sum of the first* 28 *terms of the series* –17 + (-14) + (-11) + (-8) + …

***Solution:***

a = -17

d = 3

n = 28

Sn = n/2{2a + (n - 1)d}

S28 = 28/2 {2 (-17) + (28 - 1) 3}

= 14 {-34 + 27 x 3}

= 14 { -34 + 81}

= 14 {47}

∴ S28 = 658

**EVALUATION**

(1) Find the sum of the numbers from 1 to 100.

(2) Find the sum of the first 26 terms of the A.P –18, -15, -12, -9, …

(3) Find the sum of the first 40 terms of the A.P 85, 83, 81, 79, …

(4) Find the sum of the first 21 terms of the arithmetic series 7 + 11 + 15 + 19 + …

(5) Find the sum of the first 16 terms of the series 25 + 22 + 19 + 16 + …

(6) Find the sum of the first 26 term of the series 4 + 7 + 10 + …

**PERIOD 5**

***Example 10:***

*The sum of the first* 9 *terms of an A.P is* 117 *and the sum of the next* 4 *terms is* 104.

*Find the(i) Common difference*

*(ii) First term*

*(iii)*25th *term of the A.P*

***(WAEC)***

***Solution:***

T1, T2, T3, T4 … T8, T9, T10, T11, T12, T13

117 104

S9 = 117 --------------------------(\*)

n = 9

since a sequence is normally summed from the first term

S9 + 4 = 117 + 104

∴ S13 = 221 ---------------(\*\*)

n = 13

Sn = n/2 {2a + (n - 1) d}

From (\*) above;

117 = 9/2 {2a + (9 - 1) d}

117 = 9/2 x 2a + 9/2 x 8d

117 = 9a + 36d

Divide through by 9 to have;

13 = a + 4d ----------------------(1)

From (\*\*) above;

221 = 13/2 {2a + (13 - 1) d}

221 = 13/2 x 2a + 13/2 x 12d

221 = 13a + 78d

Divide through by 13 to have;

17 = a + 6d ----------------------(2)

From equation (1) and (2)

Eqn. (1): 13 = a + 4d

Eqn. (2): 17 = a + 6d

-4 = -2d

d = -4

-2

∴ d = 2.

(ii) From equation (1) we have

13 = a + 4 x 2

13 = a + 8

a = 13 – 8

∴ a = 5

(iii)a = 5

d = 2

n = 25

Tn = a + (n - 1) d

T25 = 5 + (25 - 1) 2

= 5 + 24 x 2

= 5 + 48

T25 = 53

**EVALUATION**

(12) The sum of the first 9 terms of an A.P is 171 and the sum of the next 5 terms is 235.Find the(a) Common difference

(b) First term

(c) Sequence

(13) The sum of the first 8 terms of an A.P is 172 and the sum of the next three terms is 15. Find the (a) Common difference

(b) First term

(c) 21st term of the A.P

**PERIOD 6**

**PRACTICAL PROBLEMS INVOLVING REAL LIFE SITUATION**

***Example 11:***

*A clerk employed by a private establishment* *on an initial salary of* ~~N~~5000 *per annum. If his annual increment in salary is* ~~N~~300. *Find the total salary earned by the clerk in* 20*years.*

***Solution:***

a = 5000

d = 300

n = 20

Sn = n/2 {2a + (n - 1) d}

S20 = 20/2 {2 x 5000 + (20 - 1) 300}

= 10 {10000 + 19 x 300}

= 10 {10000 + 5700}

= 10 {15700}

∴ S20 = 157000

∴ The total amount earned in 20years is ~~N~~157000.00

**EVALUATION**

1. The value of a machine depreciates each year by 5% of its value at the beginning of that year. If its value when new on 1st January 1980 was N10,250.00, what was its value in January 1989 when it was 9years old? Give your answer correct to three significant figures.

***(WAEC) 1989***

1. New General Mathematics for Senior secondary School, Book 2, page 209, Exercise 18d, nos. 7, 13, 14, 15 and 16.

**References**

1. New General Mathematic for Senior Secondary School, Book 2, By Channon , Smith Et al.
2. Fundamental General Mathematics for Senior Secondary School, By Idode G. O

**WEEK 6:**

**Subject: Mathematics**

Class: SS 2

**TOPIC: Sequence and series 2**

**Content:**

* Geometric progression
* Calculation of first term , common ratio, and nth term, Geometric mean, sum of terms of G.P, sum to infinity,
* Practical problems involving real life situation

**PERIOD 1:**

**DEFINITION**

**GEOMETRIC PROGRESSION (G.P) OR {*EXPONENTIAL SEQUENCE*}**

Given any sequence of terms T1, T2, T3, T4, … Tn-1, Tn. If the ratio between any term and the one preceding it is constant then the sequence is said to be in geometric progression (G.P). The ratio is called the common ratio denoted by r.

i.e.

r = Tn

Tn-1 where n = 1, 2, 3, 4, …

#### Examples

(i) 1, 2, 4, 8, 16, …

Tn ⇒ 2 = 2

Tn-1 1

4 = 2

2

8 = 2

4

*The ratio is common, hence the sequence is a G.P ∴ r = 2*

(ii) 16, 8, 4, 2, …

Tn ⇒ 8 = 1

Tn-1 16 2

4 = 1

8 2

2 = 1

4 2

*The ratio is common, hence the sequence is a G.P ∴ r = 1*

*2*

(iii) 3, 7, 9, 12, …

Tn ⇒ 7 = 21/3

Tn-1 3

9 = 12/7

7

12 = 13/9

9

*The ratio is NOT common, hence not a G.P.*

(iv) 2, -10, +50, -250, …

Tn ⇒ -10 = -5

Tn-1 1

+50 = -5

-10

-250 = -5

+50

*The ratio is common, hence the sequence is a G.P ∴ r = -5*

(v) a, ar, ar2, ar3, …

Tn ⇒ ar = r

Tn-1 a

ar2  = r

ar

ar3 = r

ar2

*Has a common ratio r, hence it is a G.P*

**EVALUATION**

Which of the following sequences are G .P

1. 3,6,12,24…
2. 2,4,6,8,…
3. 5,15,45,…
4. 1,4,16,…
5. -3,-5,-7…
6. -3,6,-12,24…

###### THE nTH TERM OF A G.P

Let 5 be the first term of a G.P whose common ratio is 2. Then

The 2nd term is 5x2 = 5(2)1

The 3rd term is 5x2x2 = 5(2)2

The 4th term is 5x2x2x2 = 5(2)3

The 5th term is 5x2x2x2x2 = 5(2)4

The nth term is 5x2x2 … 2 = 5(2)n - 1

In general, the nth term of a G.P denoted by Tn, whose first term is “a” and whose common ratio is “r” is arn-1. i.e

Tn = arn - 1 where n = 1, 2, 3, 4, …

#### NOTE

The four examples below show how the formula can be used to find the nth term, n, r and a.

***Example 1:***

*Find the 8th term of the G.P*

3, 6, 12, 24, …

#### Solution

a = 3

r = 2

n = 8

Tn = arn - 1

T21 = 3(2)8 - 1

T21 = 3x27

T21 = 3x128

∴ T21 = 384

#### Example 2:

*Find the value of n given that the nth term of a G.P is* 2916 *and the first term and common ratio are* 4 *and* 3 *respectively.*

***Solution:***

Tn = 2916

r = 3

a = 4

Tn = arn - 1

2916 = 4(3) n - 1

2916 = 3n - 1

4

729 = 3n - 1

36 = 3n - 1

n-1 = 6

n = 6+1

∴ n = 7.

#### Example 3:

A ball was dropped from a height 80m above a concrete floor. It rebounded to the height of ½ of its previous height at each rebound. After how many bounces is the ball 2.5m high? ***NECO 2001***

#### Solution

#### The rebounds forms a sequence of the order

80, 40, 20, …,2.5.

a = 80

r = ½

Tn = 2.5 Tn = arn-1

2.5 = 80(½)n-1

2.5 = (½)n-1

80

5/2  = (½)n-1

80

5 = (½)n-1

2x80

1/32 = (½)n-1

(½)5 = (½)n-1

n – 1 = 5

n = 5 + 1

∴ n = 6

#### Example 4:

*Find the common ratio of an exponential sequence whose* 10th *term is* –512 *and the first term is* 1.

#### Solution

a = 1

T10 = -512

n = 10

Tn = arn - 1

-512 = 1(r)10 - 1

-512 = r9

(-512)1/9 = r

r = (-29)1/9

∴ r = -2

#### Example 5:

*Find the first term of an exponential sequence* *whose* 7th *term is* 4096 *and the common ratio is* 4.

#### Solution

T7 = 4096

n = 7

r = 4

Tn = arn - 1

4096 = a(4)7-1

4096 = a46

4096 = 4096a

a = 4096

4096

∴a = 1

**EVALUATION**

1. Find the 7th term of the G.P 1024, -512, 256,…
2. Find the 8th term of the G.P 4,- 8, 16, -32,…
3. Find the 7th term of the G.P 8748, 2916,972,…
4. Find the value of n given that -2048 is the nth term of a G.P whose first term is 64 and the common ratio is -2
5. Find the first term of a G.P, given that the 6th term is 2187 and the common ratio is 3
6. Find the common ratio of a G.P, whose first term is -729 and the 6th term is 3.
7. What is the common ratio of a G.P whose first and fourth terms are 6 and 486 respectively ?.

**PERIOD 2 and 3**

#### Example 6:

*The* 3rd *term of a G.P is* 54 *and the* 5th *term is* 486*. Find the*

* 1. *Common ratio*
  2. *First term*
  3. 7th *term of the G.P*

#### Solution

(a) T3 = 54 Tn = arn-1

n = 3 54 = ar3-1

54 = ar2 --------------- (1)

T5 = 486 486 = ar5-1

n = 5 486 = ar4  -------------- (2)

Equation (2) ÷ equation (1)

486 = ar4

54 ar2

9 = r2

r = ±√9

r = ±3

(b) Substitute in equation (1)

54 = a**(**±3)2

54 = 9a

a = 54/9

∴ a = 6

**ALTERNATIVE APPROACH**

Equation (1) ÷ equation (2)

54 = ar2

486 ar4

1 = 1

9 r2

Cross multiply

r2 = 9

r = ±√9

r = ±3

(c) a = 6

r = ±3

n = 7

Tn = arn - 1

T7 = 6(±3)7 - 1

= 6x36

= 6x729

= 4373

The example above can be reframed as shown in the example below

#### Example 7:

*The* 3rd *term of a G.P is* 54 *and the* 5th *term is 9 times the third term. Find the*

*(a) Common ratio*

*(b) First term*

(c) 7th *term of the G.P*

#### Solution:

(a) T3 = 54 Tn = arn-1

n = 3 54 = ar3-1

54 = ar2 --------------- (1)

T5 = 9x54 486 = ar5-1

= 486 486 = ar4  -------------- (2)

n = 5

Equation (2) ÷ equation (1)

486 = ar4

54 ar2

The same solutions would be obtained as shown in example above.

***Example 8:***

*If* 2, x, y, 54 *are in G.P, find x and y.*

***Solution:***

a = 2

n = 4

T4 = 54

Tn = arn - 1

54 = 2(r)4 -1

54 = 2r3

54/2 = r3

r3 = 27

r 3 = 33

∴r = 3

a = 2

r = 3

T2 = x

n = 2

Tn = arn - 1

x = 2(3)2 - 1

x = 2x3

∴ x = 6

a = 2

r = 3

T3 = y

n = 3

y = 2(3)3 - 1

y = 2(3)2

y = 2x9

∴ y = 18

***Example 9:***

*Given that* x, (x-2), (2x-1) *are in G.P. Find the*

* 1. *Value(s) of x*
  2. *Sequences*
  3. *Possible value(s) of the* 8th *term of the G.P*

***Solution:***

Since x, (x-2), (2x-1) are in G.P, the ratio between any of the terms and the one preceding must be common.

i.e (x-2) = (2x-1)

x (x-2)

Cross-multiplying

(x - 2)(x - 2) = x(2x - 1)

x2 – 2x –2x + 4 = 2x2 – x

0 = 2x2 – x – x2 + 2x + 2x – 4

0 = x2 + 3x - 4

i.e x2 + 3x - 4 = 0 (Factorize the equation)

-4x2

x2 + 4x – x - 4 = 0

x(x + 4) – 1(x + 4) = 0

(x + 4)(x - 1) = 0

x + 4 = 0 or x - 1 = 0

x = -4 or x = 1

(b) Since x = -4 and 1, we shall have two sequences

For x = -4

x, (x-2), (2x-1)

= -4, (-4 - 2), (2(-4)-1)

= -4, -6, -9

For x = 1

x, (x-2), (2x-1)

= 1, (1-2), (2x1-1)

= 1, -1, 1

(c) For x = -4 the sequence = -4, -6, -9

a = -4

n = 8

r = 3

2

T8 = -4(3/2) 8 - 1

= -4(3/2)7

= -4 x 2187

128

∴T8  = - 2187

32

For x =1, the sequence = 1, -1, 1

a = 1

n = 8

r = -1

T8 = -1(-1)8 - 1

= 1(-1) 7

= -1

###### EVALUATION

(1) The 3rd and 6th term of a geometric progression (G.P) are 48 and 142/9 respectively. Write down the first four terms of the G.P.

***SSCE, June. 1993, No9 (WAEC)***

(2) The first and third terms of a G.P are 5 and 80 respectively. What is the 4th term?

***SSCE, Nov. 1993, No 11b (WAEC)***

(3) The third and fifth terms of a geometric progression are 9/2 and 81/8  respectively.

Find the (i) Common ratio

(ii) First term.

(4) If 2, x, y, -250, … is a geometric progression, find x and y.

***(JAMB)***

(5) Given that 2, a, b, 686 are in G.P, find the value of a and b.

(6) The first three terms of a G.P are x+1, (x+4) and 2x. Find the

1. Value (s) of x

(ii) Sequence(s)

(iii) 6th term of sequences

**PERIOD 4**

GEOMETRIC SERIES

The general expression for a geometric series is given as

Sn = a + ar + ar2 + ar3 + … + arn-1 ----- (1)

Where Sn represent the sum of n terms of the series

Multiply both sides of equation (1) by r to have

rSn = ar + ar2 + ar3 + ar4 + --- + arn ---- (2)

Subtract (2) from (1) to have

Sn – rSn = a - arn

Sn (1 - r) = a(1 - rn)

Sn = a (1 - rn) ------------------ (3)

1 - r

If the numerator and denominator of equation (3) is multiplied by –1.

Sn = a( rn –1) -------------------- (4)

r - 1

If r < 1, formula (3) is more convenient

If r >1, formula (4) is more convenient

***Example 10:***

*Find the sum of the first 8 terms of the G.P* 3, 6, 12, 24, …

***Solution:***

a = 3

n = 8

r = 2

Sn = a(rn – 1) r > 1

r – 1

S8 = 3(28 – 1)

2 – 1

= 3(256 – 1)

1

= 3 (255)

S8 = 765.

***Example 11:***

*Find the sum of the first* 10 *terms of the G.P* 2, -6, 18, -54, …

***Solution:***

a = 2

n = 10

r = -3

Sn = a(1 - rn) , r < 1

1 - r

S10 = 2(1-(-3)10)

1-(-3)

= 2(1-59049)

1+3

= 2(-59048)

4

= -59048

2

S10 = -29524

# EVALUATION

(1) Find the sum of the first 9 terms of the sequence 84, 42, 21, 10½, …

(2) Find the sum of the first 10 terms of the G.P 4, 8, 16, 32, …

(3) Find the sum of the first 7 terms of the G.P 3, 9, 27, 81, …

(4) Find the sum of the first 11 terms of the G.P 5, 10, 20, 40, …

(5) If the 3rd and 7th terms of a G.P are 12 and 192 respectively, find the sum of the first 6 terms of the sequence.

**GENERAL EVALUATION**

New General Mathematic for Senior Secondary School, Book 2, page 212, exercise 18e Nos. 4,5,6.

**References**

1. New General Mathematic for Senior Secondary School, Book 2, By Channon , Smith Et al.
2. Fundamental General Mathematics for Senior Secondary School, By Idode G. O

**WEEK 2**

**SUBJECT: MATHEMATICS**

**CLASS: SS 2**

**TOPIC: LOGICAL REASONING**

**CONTENT:**

* Simple and compound statements
* Logical operation and the truth tables
* Conditional statements and indirect proofs

**SUB-TOPIC 1**

Simple and compound statements

Mathematical logic can be defined as the study of the relationship between certain objects such as numbers, functions, geometric figures etc. Statements are verbal or written declarations or assertions. The fundamental (i.e logical) property of a statement is that it is either true or false but not both. So logical statements are statements that are either reasonably true or false but not both.

**Example:** The following are logical statements;

1. Nigeria is in Africa
2. The river Niger is in Enugu
3. 2 + 5 = 3
4. 3 < 7

N.B The educator should ask the students to give their examples

**Example:** The following are not logical statements because they are neither true nor false.

1. What is your name?
2. Oh what a lovely day
3. Take her away
4. Who is he?
5. Mathematics is a simple subject (note that this statements is true or false depending on each individual, so it is not logical)

N.B educator to ask the students to give their own examples

Compound statements—When two or more simple statements are combined, we have a compound statement. To do this, we use the words: ‘and’, ‘or’, ‘if … then’, ‘if and only if’, ‘but’. Such words are called connectives. Conjunction (or ˄) of logical reasoning: Any two simple statements p,q can be combined by the word ‘and’ to form a compound (or composite) statement ‘p and q’ called the conjunction of p,q denoted symbolically as p˄q.

**Example: 1**. Let p be “The weather is cold” and q be “it is raining”, then the conjunction of p,q written as p˄q is the statement “the weather is cold and it is raining”.

2. The symbol ‘˄’ can be used to define the intersection of two sets A and B as follows;

The truth table for p˄q is given below;

|  |  |  |
| --- | --- | --- |
| p | q | p˄q |
| T | T | T |
| T | F | F |
| F | F | F |

**EVALUATION:**

Form compound statements using ‘and’. Express the compound statements in symbol form

**SUB-TOPIC 2**

Logical operation and the truth table

The word ‘not’ and the four connectives ‘and’, ‘or’, ‘if … then’, ‘if and only if’ are called logic operators. They are also referred to as logical constants. The symbols adopted for the logic operators are given below.

Logic Operators Symbols

‘not’

**‘**and**’**

‘or’ ˅

‘if … then’

‘if and only if’ ↔

When the symbols above are applied to propositions p and q, we obtain the representations in the table below:

Logic operation Representation

‘not p’ p or

‘P and q’ p˄q

‘p or q’ p˅q

‘if p then q’ pq

‘p if and only if q’ p↔q

**EVALUATION:**

Enumerate the logical operators and their symbols

**SUB-TOPIC 3**

Conditional statements and indirect proofs

Many statements especially in mathematics are of the form “if p then q”, such statements are called conditional statements or implications. The statement ‘if p then q’ means p implies q. The p part is called the antecedent (ante means before) whereas the q part is the consequent

**Example:**

1. The student can solve the problem only if he goes through the worked examples thoroughly.

Antecedent: The student can solve the problem

Consequent: He goes through the worked examples thoroughly

1. If Dayo is humble and prayerful then he will meet with God’s favour.

Antecedent: Dayo is humble and prayerful

Consequent: He will meet with God’s favour

**EVALUATION:**

Identify the antecedent and the consequent in these implicative statements

1. If I travel then you must teach my lesson
2. If you person well in your examinations then you will go on holidays
3. If London is in Britain then 12 is an even number
4. If the bus come late then I will take a motorcycle
5. If a & b are integers then ab is a rational number

**Converse statements**: The converse of the conditional statement “if p then q” is the conditional statement “if q then p” i.e the converse of pq is q p

Example;

Let p be ‘Obi is a boy’ and q be ‘3 + 3 = 4’ and so pq is the statement ‘if Obi is a boy then 3 + 3 = 4’. The converse of the statement (q p) is the statement ‘if 3 + 3 = 4 then Obi is a boy’

(students should give more examples)

**Inverse statements:** The inverse of the conditional statement “if p then q” is the conditional statement “if not p then not q”. i.e the inverse of pq is pq

**EVALUATION:**

1. Write down the inverse of each of the following statements
2. If Mary is a model then she is beautiful
3. If Ibadan is the largest city in the west Africa then it is the largest city in Nigeria
4. If the army misbehaves again he will be demoted
5. Write down the converse of each of the following
6. If he sets a good, he will get a good fellowship
7. If it rains sufficiently then the harvest will be good
8. If the triangles are congruent then the ratios of their corresponding lengths are equal

**WEEK 3**

**SUBJECT: MATHEMATICS**

**CLASS: SS 2**

**TOPIC: LINEAR INEQUALITIES**

**CONTENT:**

* Revision of linear inequalities in one variable
* Solution of inequalities in two variables
* Range of values of combined inequalities

**SUB-TOPIC 1**

Number line can be used to show the graph of inequalities in one variable. Symbols commonly used for inequalities include;

Steps taken in solving inequalities is similar to that of equations with few exceptions such as

1. Reversing the inequality sign when both sides are multiplied (or divided) by negative quantity. i.e if 2 < 5 then -2 > -5
2. Reversing the inequality sign when reciprocals are taken

i.e if

**Example;** solve (a)

**solution;**

0 1 2 3 4

Notice that the right end point x=4 is not part of the solution so the circle above is not shaded.

(b) solve the inequality ;

**Solution;** To clear the fraction, multiply through by the LCM of the denominators i.e 12

-2 -1 0 1 2

Notice that the left end point, is part of the solution, so small circle above is shaded.

**Example;** Find the range of values of x which satisfy (WAEC)

Solution;

-4 -3 -2 -1 0 1 2 3 4

**EVALUATION:**

1. Solve the inequality and represent your result on a number line;

1. Find the three highest whole number that satisfy
2. Solve and show on number line the values of x which satisfy
3. NGM for SS2 Ex. 10a nos 15, 18, 24. Ex. 10c nos 3, 4, 5

**SUB-TOPIC 2**

Solution of inequalities in two variables

For linear inequalities in two variables, first draw the corresponding straight line. Inequalities in two variables are usually plotted on plane (the Cartesian coordinate plane)

Example: Show the expression on a graph.

Solution; first put y on one side of the inequality i.e

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | -3 | -1 | 0 | 3 |
|  | -1 | 0 | 1 | 2 |

Then draw the corresponding line

This line divides the plane into two. To find the side with the solution, we select and try out a pair of points. E.g

For

Hence, the solution set is in the region above the line

The upper part of the graph shaded satisfies the inequality

**EVALUATION:**

1. Shade the region common to
2. Show on a graph the region that contains the set of points for which
3. Shade the region that satisfy the following

Now we shall consider range of values of combined inequalities.

**Example:** if what range of x satisfies both inequalities?

Solution: solving the inequalities separately we obtain respectively.

**EVALUATION:**

1. What range of satisfy both
2. Find the range of values of such that
3. Express the inequality in the form where are both integers

**WEEK 4**

**SUBJECT: MATHEMATICS**

**CLASS: SS 2**

**TOPIC: LINEAR INEQUALITIES**

**CONTENT:**

* Graph of linear inequalities in two variables
* Maximum and Minimum values of simultaneous linear inequalities
* Application of linear inequalities in real life
* Introduction to linear programming

**SUB-TOPIC 1**

Graph of linear inequalities in two variables: We shall consider simultaneous inequalities.

**Example:** Show on a graph the region that contains the solution of the simultaneous inequalities

**Solution:** In each case put on one side of the inequality

We shall draw the lines

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | -2 | 0 | 2 | 3 |
|  | 3.3 | 2 | 0.7 | 0 |
|  | -2 | 2 | 6 | 8 |

y-axis

8

7 y=2+2x

6

5

4

3

2

1

-3 -2 -1 0 1 2 3 4 x-axis

-1

-2 y=

-3

-4

Points are in the solution set for the three inequalities.

The shaded portion is the required region. The integral values of x & y that satisfy the inequalities simultaneously are (-1,0), (0,0), (0,1), (0,2), (1,0), (1,1), (2,0), (3,0)

**EVALUATION:**

1. Shade the region defined by;
2. Show on a graph the region which contains the solutions of the simultaneous inequalities

1. Find the region common to show the region on a graph

**ASSIGNMENT:**

NGM for SS2, Ex. 10e nos 1, 2 and 5c

**SUB-TOPIC 2 & 3**

Maximum and Minimum values of simultaneous linear inequalities & Application of linear inequalities in real life.

In solving simultaneous inequalities involving variables x & y, the expression x+y=n is called the objective function. Linear programming usually involves either maximizing or minimizing the function x+y=n. These problems are sometimes called minimax problems.

**Example:** A manufacturer has 120kg and 100kg of wood and plastic respectively. A product requires 2kg of wood and 3kg of plastic. Product requires 3kg of wood and 2kg of plastic. If A sells for #3500 and B for #5000. How many must be made to obtain the maximum gross income?

Solution:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Wood (kg/unit) | Plastic (kg/unit) | # per kg |
| Product A | 2 | 3 | 3500 |
| Product B | 3 | 2 | 5000 |

Suppose there is x number of product A and suppose there is y number of product B

For the first inequality we shall draw line 2x + 3y = 120

If x=0, y=40.

**WEEK 5**

**SUBJECT: MATHEMATICS**

**CLASS: SS 2**

**TOPIC: ALGEBRAIC FRACTIONS**

**CONTENT:**

* Simplification of fractions
* Operation in algebraic fractions
* Equation involving fraction
* Substitution in fraction
* Simultaneous equation involving fraction
* Undefined value of a function

**SUB-TOPIC 1**

Simplification of fractions

An algebraic fraction is a part of a whole, represented mathematically by a pair of algebraic terms. The upper part is called the numerator while the lower part the denominator. To simplify algebraic fractions, we need to factorize both the numerator and the denominator.

Example 1: Reduce the following to their lowest term

Solution:

Cancel the common factors i.e

.

(b.)

(c.)

(d.)

**EVALUATION:**

Simplify the following fractions

**SUB-TOPIC 2**

Operations in algebraic fractions are the process of adding, subtracting, multiplying and dividing algebraic fractions

**Addition and Subtraction algebraic fractions**

**Examples;** Simplify the following

**Solution:**

Express the two fractions as a single fraction by taking LCM

(b.)

Take the LCM and then express a single fraction

(c.)

The LCM is the product of the denominator of the three terms

(d.)

**EVALUATION:**

Simplify the following expressions to its lowest terms

**Multiplication and division of fractions**

In multiplication and division of algebraic fractions, we need to factorize both the numerator and the denominator fully and then divide both the numerator and denominator by common factor(s)

**Examples:**



**Solution:**

.

.

(b.)

(c.)

Re-writing the question and factorise each fraction fully, we have

After thorough and correct factorization we then cancel factors accordingly

**EVALUATION:** Simplify the following to its lowest term

**SUB-TOPIC 3**

**Substitution in fraction**

**Example 1:**

Given , evaluate

Solution: divide both numerator and denominator by

⇒

Substitute in the algebraic expression

.

Or we can also check by putting

⇒

Example 2:

If in terms of d

Solution:

Substitute for in the given expression

⇒ .

Multiply the numerator and the denominator by , we obtain

.

.

.

**EVALUATION:**

1. Given p:q = 9:5, evaluate
2. If , express in terms of
3. If evaluate
4. If , then express in terms of

**SUB-TOPIC 4**

**Equation involving fraction**

**Example 1:** Solve the equation;

Solution: on cross multiplying, we have

.

.

Collecting like terms

,

,

,

**EVALUATION:**

Solve the following equation

**SUB-TOPIC 5**

**Simultaneous equation involving fractions**

**Examples;**

1. Solve the simultaneous equation

**Solution;**

...........(ii)

Multiply each term of equation (i) by 10 and multiply each term of equation (ii) by 4

⇒

Subtracting equation (iv) from (iii), we have

,

Divide both sides by 3

,

Substitute 4 for y in equation (iv)

1. Solve the equation;

**Solution:**

.............(ii)

Multiply each term in equation (i) by 6 and equation (ii) by 12

Multiply each term in equation (2) by 12

Adding equations (iii) & (iv)

Substitute 5 for x in equation (iv)

Divide both sides by

**EVALUATION:**

Solve the following pairs of equations

**SUB-TOPIC 6**

**Undefined value of a function**

An algebraic fraction whose denominator is equal to zero is said to be undefined. If an expression contains an undefined fraction, the whole expression is undefined. For instance, will be undefined if the value of is

When then; , but division by zero is impossible. Therefore the fraction is **undefined.** Below is the table of values and corresponding graph of the function values of x ranges from -3 to 5

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
|  | -0.4 | -0.5 | -0.67 | -1 | -2 | 0 | 2 | 1 | 0.67 |

Notice that: (i) As the values of x approaches 2 from below the value of decreases rapidly.

(ii) As the value of x approaches 2 from above, the value of increases rapidly.

When

Division by zero is impossible. The fraction is said to be undefined when

The table of values and the graph clearly shows that is undefined when

**Examples;**

Find the values of for which the following fractions are not defined.

**Solution:**

1. is undefined when , if then

The fraction is not defined when

1. is undefined when

Which implies that ,

**EVALUATION:**

1. If k is a constant not equal to zero. Find the value(s) of x for which the expression is undefined
2. Find the values of x for which the following expressions are undefined.

**WEEKEND ASSIGNMENT:**

NGM for SS2 pages 200 – 201, exercise 17g (miscellaneous practice questions 1 - 20)

**WEEK 6**

**SUBJECT: MATHEMATICS**

**CLASS: SS 2**

**TOPIC: CHORD PROPERTY**

**CONTENT:**

* Riders based on the circle theorems include
* Angles subtended by chords in circle
* Angles subtended by chords at the centre
* Perpendicular bisectors of chords
* Angles in alternate segments

**SUB-TOPIC 1**

**Angles subtended by chords in circle**

The word chord is a straight line joining any two points such as A and B on the circumference of a circle. The chord divides the circle into two parts called the segments (minor and major)

Major Arc

major segment

chord

minor segment

Minor Arc

The larger part of the circle is called the major segment while the smaller part --- the minor segment. Each of these parts is called the alternate segment of the other.

Note: A major segment has a major arc while a minor segment a minor arc.

A circle is the set of all points at a constant distance from a fixed point in a plane. The fixed point is the centre of the circle, the distance from the fixed point (is constant), is called the radius.

It will be noted that it is the chord that subtends (project out) angles viz:

Q

P R

A B

From the diagram, P,Q and R are points on the circumference of a circle. are angles subtended at the circumference by the chord AB or by the minor arc AB. are all angles in the same major segment APQRB.

Similarly, from the diagram below

**A B**

X Y

.A are angles subtended by the chord AB or by the major arc AB in the minor segment AXYB or the alternate segment.

**SUB-TOPIC 2**

**Angles subtended by chords at the centre**

**Examples:**

**THEOREM:** A straight line drawn from the centre of the circle to the middle point of a chord which is not a diameter, is at right angle

O

A D B

Given: A chord AB of a circle with centre O, is the mid-point of AB such that AD = DB

To prove:

Construction: join OA and OB

Proof: (radii of the circle)

(Given)

is common

Hence

But

⇒

**THEOREM:** Equal chords of a circle are equidistant from the centre of the circle.

A D

M N

B C

**Given:** chord AB = chord DC

**To prove:**

**Construction:** join

**Proof:** In

OA = OD (radii)

Converse: chords that have the same distance (i.e equidistant) from the centre of the circle are of the same length. If , then

**Example:** A chord of length 24cm is 13cm from the centre of the circle. Calculate the radius of the circle

**Solution:**

O

r

P M Q

From the diagram,

In

.

= 169 + 144

= 313

, r = = 17.69cm

**EVALUATION:**

1. NGM for SS2 page 27, Ex. 2a nos 1, 2, 7
2. A chord is 5cm from the centre of a circle of diameter 26cm.Find the length of the chord. (WAEC)
3. Find marked angles in the following if point O is the centre of the circles:

P

r

A B R

**SUB-TOPIC 3**

**Perpendicular bisectors of chords**

This talks of line(s) that divides another line into two equal parts.

**THEOREM**: A straight line drawn from the centre of a circle perpendicular to a chord bisects the chord.

O

A D B

**Given:** A chord AB of a circle with centre O and

**To prove:**

**Construction:** join OA and OB

**Proof:** In

(given)

OD is common

**Example;**

XYZ is an isosceles triangle inscribed in a circle centre O. XY = XZ = 20cm and YZ = 18cm. calculate to 3s.f

1. The altitude of XYZ
2. The diameter of the circle

**Solution:** X X

A B

Y Z Y Z

In

.

(XQ) =

= 17.9cm

(b.) is the diameter of the circle , radii =

In ,

In

But diameter,

**Example:**

The diagram below shows two parallel chords AB and CD that lie on opposite sides of the centre O of the circle. AB = 40cm, CD = 30cm and the radius of the circle is 25cm. Calculate the distance h between the two chords

A E B

H O

C F D

**Solution:**

Similarly,

In by Pythagoras’ theorem,

.

.

In

But,

**EVALUATION:**

1. A chord 26cm long is 10cm away from the centre of a circle. Find the radius of the circle.
2. The diameter of a circle is 12cm if a chord is 4cmfrom the centre, calculate the length of the chord.
3. Find the distance between chord AB and CD in the following diagrams

1. A C (b) A B

B D C D

**SUB-TOPIC 4**

**Angles in alternate segments**

Recall: The chord that passes through the centre of the circle is called diameter and is the largest chord in a circle.

A segment is a region bounded by a chord and an arc lying between the chord’s end point.

The chord that is not a diameter divides the circle into two segments -- a major and a minor segment.

But, a tangent to a circle is a straight line that touches the circle at a point.

Thus;

**THEOREM:** An angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment

D

E

C

B

P A Q

**Given:** A circle with tangent PAQ at A and chord AC dividing the circle into two segments AEC and ABC. Segments AEC is alternate to

**To prove:** <QAC = <AEC and <PAC

= <ABC

**Construction:** Draw the diameter AD. Join CD

**Proof:** From the lettering in the above,

Also, <ACD = 90 (angle in a semi-circle)

In

<ACD = 180 (sum of angles in a )

Subtracting from equations (i) and (ii)

Also, B is a point in the minor segment.

< PAC + < CAQ = 180 (angles on a straight line)

< PAC + = 180

< PAC = 180 –

= 180 (proved )

< PAC = < ABC (opposite angles of a cyclic quadrilateral)

**Example:**

is a tangent to circle QPS. Calculate < SQX

X

S Q

P Z

**Solution:**

In < SPQ = 180 – (55 + 48)

= 180 – 103

= 77

.< SQX = 77 (angles in alternate segment)

**Example:** N

Y

Z

L X M

From the above, are tangents to the circle with centre O. Find X

**Solution:**

<XYZ = 2 (angles in alternate segment)

<ZYN + 100 = 180 (angles on a straight line)

<ZXY = 80 (angles in alternate segment)

<XZY = 100 – 2x (angles in alternate segment)

<MXY = <XZY = 100 – 2x (angles in alternate segment)

.<MXY = <MYX =100 – 2x

. is an isosceles triangle

.35 + 2(100 –2x) = 180 (sum of angles in a )

2(100 –2x) = 180 – 35

200 – 4x = 145

4x = 55

.

**EVALUATION:**

1. Find the marked angles indicated by letters in the following diagrams

1. MP is a tangent to the circle LMN at M. If the chord LN is parallel to MP, show that the triangle LMN is isosceles. (WAEC)

N

P

L M

1. Prove that the angle between a tangent and a chord is equal to the angle subtended by the chord in the alternate segment.
2. MNP is a tangent to the circle ABN at N. ABC is a straight line and NC bisects <BNP. Find x

A B C

M N P

**GENERAL ASSIGNMENT:**

NGM for SS2 page 27, Ex. 2a nos 6, 7, 8, 9 and 10

**WEEK 7**

**CLASS: SS 2 SECOND TERM**

**TOPIC: CIRCLE THEOREM**

SUB-TOPIC: PROOF OF (i) The angle which an arc subtends at the centre is twice the angle it subtends at the circumference.

(ii) Angles in the same segment.

(iii) Angle in a semi-circle.

(iv) Cyclic Quadrilateral.

(v) Tangent to a circle.

**Period 1**

The angle which an arc (or a chord) of a circle subtends at the centre of the circle is twice the angle which it subtends at any point on the remaining part of the circumference.

Note: An arc of a circle is any connected part of the circle’s circumference.

A chord which is not a diameter divides the circle into two arcs- a major and a minor arc.

Given: An arc AB of a circle with ‘O’ and a point ‘P’ on the circumference.

To Prove: A

Construction: Join and produce the line to a point D

Sketch:

P P

X1 y1 A x2 X1 y1

o O y2 B

X2 y2

A D B D

**(i) (ii)**

P

O X1 y1

D

**(iii)** X2 y2

A B

Proof: since (radii in the same circle)

(base angles of isosceles AP)

AD = (exterior angle of AP)

AD = 2 (since )

Similarly, BOD = 2

In (a) acute/obtuse AOB = AOD + BOD

In (b) reflex AOB = AOD + BOD

= 2 + 2

= 2()

= 2APB

In (c) AOB = AOD – BOD

= 2

= 2()

= 2APB

AOB = 2APB (in all cases)

(2) in the diagram below, O is the centre of the circle ACB. If <CAO = 26⁰ and <AOB = 130⁰, calculate (a) <OBC and (b) <COB (WAEC)

C

α α

O

260 1300

A B

**Solution:**

ACB =

= 65⁰

= α + α

α =

= 32.5⁰

AOC = 180 – (26 + 32.5)

= 180 – 58.5

= 121.5⁰

COB = 360 – (130 + 121.5) (angle at a point)

= 360 – 251.5

= 108.5⁰

∴ OBC = 180 – (108.5 + 32.5)

= 180 – 141

= 39⁰

(3) Given a circle with centre O while A,B and C are points on the circumference. Find <ABC, if the obtuse <AOC = 125⁰

B

A C

1250

O

**Solution:**

Reflex AOC = 360 – 125 (angle at a point)

= 235⁰

∴ ABC = (angle at the centre is twice the angle at the circumference)

= 117.5⁰

**EVALUATION**

1. Find the lettered angles in each of the figures below;

(a) K 300 (b)1200

2000 O y

J O

i x

z

1. In the diagram, ABCD is a circle centre O. AC and BD intersect at right angles at K. Angle COD is 130⁰, calculate angles (i) DAC

(ii) ADB

(iii) AOB (WAEC)

A

B K

O

1300 D

C

1. (a) Prove that the angle which an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference.
2. In the diagram below, O is the centre of the circle <OQR = 32⁰ and <MPQ = 15⁰

Calculate: (i) <QPR

(ii) <MQO (WAEC)

P

150

M S

O

Q 320 R

**Period 2**

Angles in the same segment of a circle are equal.

Given: points A,B and C on the major segment of a circle ABCDE with centre O.

To Prove: <EAD = <EBD = <ECD

Construction: Join EO; DO

B

A C

P q r

O

E D

**Proof:**

EOD = 2p (angle at the centre is twice angle at the circumference)

EOD = 2q (angle at the centre is twice angle at the circumference)

EOD = 2r (angle at the centre is twice angle at the circumference)

⇒ p = q = r

∴ EAD = EBD = ECD

(2) The diagram below shows a circle ABCD in which <DAC = 55⁰ and <BCD = 100⁰, find BDC.

A B

550

D 1000 C

**Solution:**

<CAD = <CBD = 55⁰ (angles on the same segment)

∴ <BDC + <CBD + <BCD = 180⁰ (sum of the angles of a triangle)

⇒ <BDC + 55⁰ + 100⁰ = 180⁰

⇒ <BDC = 25⁰

(3) In the diagram below, PQRS is a circle if /PT/ = /QT/ and <QPT = 70⁰, calculate < PRS? (WAEC)

P Q

700

S R

In PQT, PT = TQ (isosceles triangle)

∴ QPT = PQT = 70⁰

But PQ = SR common chord

SRT = QPT = 70⁰ (alternate angle)

**EVALUATION**

1. Find the lettered angles in each of the figures below;
2. (b)

M e I h f

O d

N 15 50 40 g

550

1. The diagram below is a circle with its centre at O. Find the value of (a)

3x+3

y-8

60

1. P,Q,R and S are points on the circle. If <PSQ = 30⁰, <PRS = 50⁰ and <PSQ = 20⁰, what is the value of

P

Y x Q

30

200

S 50 R

**Period 3**

Angle in a semi-circle

**Given:** PQ is the diameter of a circle with c entre O and R is any point on the circumference.

**To Prove:** PRQ = 90⁰

**Construction:** PR, RQ

R

P Q

Proof:

<POQ = 2PRQ (angle at the centre is twice that at the circumference)

But POQ = 180⁰ (angle on a straight line)

∴ 2PRQ = 180⁰

PRQ =

∴ PRQ = 90⁰

(2) In the diagram, O is the centre of the circle. If <BAC = 55⁰, find the value of <ACB

B

A C

**Solution:**

<ABC = 90⁰ (angle in a semi-circle)

<ABC + <ACB + <BAC = 180⁰ (sum of angles in a triangle)

⇒ 90⁰ + <ACB + 55⁰ = 180⁰

<ACB + 145⁰ = 180⁰

<ACB = 180 – 145

< ACB = 35⁰

(3) Find the values of the lettered angles in the figure below;

B

X 60

y

A D O C

**Solution:**

<ABC = 90

∴ X = 90 – 60

= 30

In ABD, <ADB = 90 (perpendicular bisector)

∴ x + y + 90 = 180

30 + y + 90 = 180

y = 180 – 120

y = 60⁰

**EVALUATION**

1. Find the values of the lettered angles in the figures below;
2. 54

64 a

O



30

y O x



630

c

O

1. In the diagram, AB is the diameter. <ABC = (5x + 3)⁰ and <BAC = (5y + 7)⁰. Express y in terms of x

C

5y+7 5x+3

A B

**Period 4**

Cyclic Quadrilateral

1. Quadrilateral is a four sided plane shape
2. A cyclic quadrilateral is a quadrilateral that is enclosed in a circle such that the four vertices touch the circumference of the circle.

Note: the four points where the vertices touch are referred to as concyclic points.

P Q

S R

1. Theorem:

The opposite angles in a cyclic quadrilateral are supplementary.

Note: Two angles are supplementary if their sum is 180 and complementary if their sum is 90.

Given: A cyclic quadrilateral ABCD in a circle with centre O.

To prove: <BAD + < BCD = 180

Construction: Join OB, OD

B

A a

2c O 2a C

D

Proof: Using letters in the diagram, Let <BAD = a

Reflex BOD = 2a (angle at the centre is twice the angle at the circumference)

Let < BCD = c

Obtuse BOD = 2c (angle at the centre is twice the angle at the circumference)

But 2a + 2c = 360 (angle at a point)

⇒ 2(a + c) = 360

⇒ a + c =

∴ a + c = 180⁰

1. Theorem

The exterior angle of a cyclic quadrilateral is equal to the interior opposite angles. Using the letters in the diagram,

P Q

a1 b1

d1 c1 a2 T

S b2

U

Given: A Cyclic quadrilateral PQRS

To prove:

Construction: Produce SR to T and PS to U.

Proof:

(opposite angles of a cyclic quadrilateral)

(angles on a straight line)

⇒

Similarly;

(opposite angles of a cyclic quadrilateral)

(angles on a straight line)

⇒

**EVALUATION**

1. Find the lettered angles in each of the figures below;

(a) P

a

M b O N

1150

Q

Q

(b)

R q

150

S 420 P

T U

1. O is the centre of the circle PQRST. If <SPT = 42⁰, <PST = 55⁰ and <PSQ = 15⁰, Find <QRS.

P

420

T Q

550150

S R

1. Find angle h in the diagram below;

650

750

h

**Period 5**

Tangent to a circle

The tangent to a circle is a straight line drawn to touch the circle at a point. The point where the line touches the circle is referred to as the point of contact.

A secant is a straight line that cuts a given circle into two clear points

secant

Point of contact

Note:

1. A tangent to a circle is perpendicular to the radius drawn to its point of contact.
2. The perpendicular to a tangent at its point of contact passes through the centre of the circle.

Theorem:

Two tangents drawn to a circle from an external point are equal in length.

Given: An exterior point T of a circle with centre O. TY and TX are tangents to the circle at X and Y.

X

O

T

Y

To Prove: /TX/ = /TY/

Construction: Join TX, TO and TY

Proof: In triangles TXO and TYO

TXO = TYO = 90 (tangent perpendicular to radius)

/XO/ = /YO/ (radius)

/TO/ = /TO/ (common)

∴ /TX/ = /TY/

1. AB and AC are tangents from a point A to a circle centre O. If <BAC = 54⁰, find the value of X

O X

540

Solution:

ABO = ACO (tangents to a circle from an external point are equal)

ABO = ACO = 90 (tangents perpendicular to radius)

∴ ABO + ACO + BAC + X = 360 (sum of angles in a quadrilateral)

⇒ 90 + 90 + 54 + X = 360

⇒ 234 + X = 360

⇒ X = 360 – 234

∴ X = 126⁰

1. Calculate PRQ

R

P

O

880

Q T

Solution:

PTQ = 88⁰

Join PO and QO

OP and OQ are radii

TQO = TPO = 90 ( radii perpendicular to tangent)

∴ OPT + OQT = 180

PTQ + QTP = 180

QOP = 180 – 88

= 92⁰

But QRP = ½ (QOP) (angle at centre is twice angle at the circumference)

= ½ (92)

= 46

∴ PRQ = 46⁰

**EVALUATION**

1. Calculate the values of the marked angles below;

O 28

x R

(b)

O

45 45

(C)

18

30 y

1. TS is a tangent to a circle PQRS. If /PR/ = /PS/ and PQR = 117⁰, calculate RST (WAEC)

Q

R P

T S

GENERAL ASSIGNMENT

1. PQRT is a circle. /ST/ = /RS/ and TSR = 51⁰, find POR (JAMB)

S

O T

P R

Q

1. AB and CB are tangents to the circle. Given that CBA = 54⁰, calculate <ADC

(NECO)

A

D 54o **B**

C

1. TP is a tangent to the circle TRQ with centre O. if <TPO = 28⁰ and <ORQ = 15⁰. Find ( a) <RQT (b) <QTO (NECO)

T

O

15 28

R P

Q

1. PQRST lie on the circumference of the circle with centre O. The chords PS and RT intersect at V and the chords PT and RS produced meet at X as shown below;

T

P X

O V

Q

S

R

Given that the obtuse POR = 4 PXR

Prove that: (a) SVT = 3 PXR , (b) PSR = PQR (London G.C.E)

1. O is the centre of the circle. <OQR = 32⁰ and <TPQ = 15⁰, Calculate (a) <QPR (b) <TQO (WAEC)

P

150

T S

O

320

Q R

READING ASSIGNMENT

1. New General mathematics for S.S 2, page 124 to 133
2. Nelson functional mathematics for S.S 2, page 168 to 191

**WEEK 8:**

**Subject: Mathematics**

Class: SS 2

**TOPIC: TRIGONOMETRY (Sine and Cosine Rule)**

**Content:**

* Derivation of sine rule
* Application of sine rule
* Derivation of cosine rule
* Application of cosine rule

**PERIOD 1:**

**SINE RULE**

Given any triangle ABC (acute or obtuse), with the angles labeled with capital letters A, B, C and the sides opposite these angles labeled with the corresponding small letters a, b, and c respectively as shown below.

C

C

b a b a

A c B A c B

The sine rule states that;

a = b = c

sinA sinB sinC

OR

sinA = sinB = sinC

a b c

**PROOF OF THE RULE**

### **Using Acute – angled triangle**

C

b h a

A c B

***Given:*** Any ∆ABC with B acute.

***To prove:*** a = b = c

sinA sinB sinC

***Construction:*** Draw the perpendicular

from C to AB.

***Proof:*** *Using the lettering in the diagram above.*

sinA = h

b

h = bsinA --------------------- (1)

sinB = h

a

h = asinB ---------------------- (2)

From equation (1) and (2)

bsinA = asinB

∴ a = b

sinA sinB

Similarly, by drawing a perpendicular from B to AC

a = c

sinA sinC

a = b = c

sinA sinB sinC

***Q.E.D***

### ***Using Obtuse – angled triangle***

C

b

a h

A c B

***Given:*** any ∆ABC with B obtuse

***To Prove:*** a = b = c

sinA sinB sinC

***Construction:*** Draw the perpendicular

from C to AB produced.

***Proof:*** *With the lettering in the diagram.*

sinA = h

b

h = bsinA -----------------(1)

sin(180 – B) = h but sin (180-θ) = sinθ

a

∴ sinB = h

a

h = asinB ----------------(2)

From equation (1) and (2)

bsinA = asinB

a = b

sinA sinB

Similarly, by drawing a perpendicular from A to CB produced.

b = c

sinB sinC

a = b = c

SinA sinB sinC

***Q.E.D***

## APPLICATION OF SINE RULE

The sine rule is used for solving problems of triangle, which are NOT right – angled, and in which either two sides and the angle opposite one of them are given or two angles and any side are given.

***Example 1:***

*In ΔABC, a = 9cm, B = 1100, b = 13cm. Solve the triangle completely.*

***Solution:***

The diagram representing the information above is given below as

C

b = 13cm a = 9cm

1100

A c B

***Using sine rule***

a = b

sinA sinB

9 = 13

sinA sin1100

9sin 1100 = 13sinA

sinA = 9sin700

13

sinA = 0.6506

A = sin-1 0.6506

A = 40.60

∴ A ≈ 410 (*nearest degree*)

To find angle C

A + B + C = 1800 *[sum of <s in a Δ]*

410 + 1100 + C = 1800

C = 1800 - 1510

∴ C = 290

To find side c, use sine rule

a = c

sinA sinC

9 = c

sin41 sin29

c = 9sin29

sin41

c = 6.65cm

∴ c = 6.7cm

***Example 2:***

*In ΔPQR, given that P = 500, Q = 600,*

*r = 7.5cm. Find (i) p (ii) q*

***Solution:***

R

q p

500 600

P 7.5cm Q

(i) P + Q + R = 1800 *[sum of <s in a ∆]*

500 + 600 + R = 1800

R = 1800 - 1100

R = 700

***Using sine rule***

r = p

sinR sin P

7.5 = p

sin700 sin500

p = 7.5sin500

sin700

p = 6.11cm

∴ p ≈ 6cm

(ii)  ***Using sine rule***

r = q

sinR sinQ

7.5 = q

sin700 sin600

q = 7.5 sin 600

sin700

q = 6.9cm

∴ q ≈ 7cm

**EVALUATION.**

Find the missing sides and angles of the following triangles. Calculate all angles to the nearest degree and all sides to 1 decimal place.

1. ΔABC, given that B = 680, b = 27m and a = 22m.
2. ΔPQR, given that Q = 1210, q = 57km and r = 17km.
3. ΔABC, given that C = 270, c = 7cm and b = 13cm.

**PERIOD 2:**

**COSINE RULE**

Given any triangle ABC (acute or obtuse), with the angles labeled with the capital letters A, B, C and the sides opposite these angles labeled with the corresponding small letters a, b, and c respectively as shown below

C C

b a

b a

A c B B                                                    A      c

**The cosine rule states that**

a2 = b2 + c2 – 2bc cosA

b2 = a2 + c2 – 2ac cosB

c2 = a2 + b2 – 2ab cosC

**PROOF OF THE RULE**

***Using acute – angled triangle***

C

b a

h

A c- x D x B

c

***Given:*** Any ΔABC with B acute.

***To prove:*** b2 = a2 + c2 – 2ac cos B

***Construction:*** Draw a perpendicular

from C to AB.

***Proof:*** *With the lettering in the diagram.*

b2 = (c - x)2 + h2 *(Pythagoras)*

= c2 – 2cx + x2 + h2

But in Δ BCD, a2 = x2 + h2

∴ b2 = c2 – 2cx + a2 ----------(1)

In ΔBCD,

cosB = x

a

∴ x = a cos B

From Eqn (1)

b2 = c2 + a2 – 2cx

b2 = c2 + a2 – 2ca cos B

***Q.E.D***

***Using obtuse – angled triangle***

C

b

a h

A c B x D

c + x

***Given:*** Any ΔABC with B obtuse

***To prove:*** b2 = a2 + c2 – 2ac cos B

***Construction:*** Draw the perpendicular

from C to AB produced.

***Proof:*** *With the lettering in the diagram.*

b2 = (c + x)2 + h2

= c2 + 2cx + x2 + h2

But in ΔBCD

a2 = x2 + h2 *(by Pythagoras)*

∴ b2 = c2 + 2cx + a2

ie b2 = a2 + c2 + 2cx -------- (1)

In ΔBCD, cosB = x

a

cos (180 – B) = x

a

-cosB = x

a

∴ x = -acosB

From Eqn (1)

b2 = a2 + c2 + 2c(-acosB)

∴ b2 = a2 + c2 – 2accosB

***Q.E.D***

Similarly, a2 = b2 + c2 – 2bccosA

c2 = a2 + b2 – 2abcosC

**APPLICATIONS OF COSINE RULE**

Cosine rule can be used for solving problems involving triangles, which are not right–angled, in which two sides and the angle between the two sides are given i.e. two sides and the included angle.

Secondly, the formula can be used to find the angles of a triangle when the three sides of the triangle are given.

**USING COSINE RULE TO FIND THE MISSING SIDE OF A TRIANGLE**

***Example 3:***

*In Δ ABC, given that A = 650, b = 9cm and c = 12cm, Find a.*

***Solution:***

C

9cm a

650

A 12cm B

***Using cosine rule***

a2 = b2 + c2 – 2bccosA

= 92 + 122 – 2x9x12cos65

= 81 + 144 – 216cos65

= 225 – 216 x 0.4226

= 225 – 91.28

= 133.72

a = √133.72

∴ a = 11.56cm.

***Example 4:***

*Find the value of q in the figure below.*

R

q

5m

1120

P 7m Q

***Solution:***

***Using cosine rule***

q2 = p2 + r2 – 2prcosQ

= 52 + 72 – 2x5x7cos1120

= 25 + 49 – 70[-cos(180 – 112)]

= 74 – 70(-cos 68)

= 74 + 70cos68

= 74 + 70 x 0.3746

= 74 + 26.222

= 100.222

q = √100.22

∴ q = 10.01

∴ q ≈ 10m

***Example 5***

*In Δ ABC, B = 1300, a = 4.62cm and c = 6.21cm, Calculate b.*

***Solution:***

A

b

6.21cm

1300

B 4.26cm C

***Using cosine rule***

b2 = a2 + c2 – 2ac cos B

= 4.622+6.212–2x4.62x6.21cos1300

= 21.34+38.56–57.38[-cos180–130]

= 59.9 - 57.38 [-cos 50]

= 59.9 + 57.38 x 0.6428

= 59.9 + 36.88

= 96.78

b2 = √96.78

∴ b = 9.8cm.

**EVALUATION**

Solve the following questions and approximate all answers to 1 decimal place.

(1) In ΔABC, B = 530, c = 45km and

a = 63km. Find b.

(2) In ΔPQR, Q = 1110, r = 47km and p =

39km. Find q.

(3) In ΔABC, B = 870, a = 25m and c =

19m. Find b.

(4) In ΔABC, B = 1420, a = 33km and c =

27km. Find b.

**PERIOD 3:**

**USING COSINE RULE TO CALCULATE ANGLES**

Cosine rule can also be used to calculate the angles of a triangle when the three sides are given. This is done by making the cosine of the desired angle the subject of the formula.

E.g. If

a2 = b2 + c2 – 2bc cos A

2bccosA = b2 + c2 – a2

cosA = b2 + c2 – a2

2bc

Similarly, cosB = a2 + c2 – b2

2ac

and cosC = a2 + b2 – c2

2ab

This formula is used to calculate the angles of a triangle when all the three sides of the triangle are given.

***Example 7:***

*Find the angles of the Δ ABC given that a = 7cm, b = 6cm and c = 5cm.*

***Solution:***

C

6cm 7cm

A 5cm B

To find angle A,

cosA = b2 + c2 – a2

2bc

= 62 + 52 - 72

2x6x5

= 36 + 25 – 49

60

cosA = 0.2000

A = cos-1 0.2000

∴ A = 78.50 ----------------- (1)

To find angle B,

cosB = a2 + c2 – b2

2ac

= 72 + 52 - 62

2 x 7 x 5

= 49 + 25 – 36

70

cosB = 0.5429

B = cos-1 0.5429

∴B = 57.10 -----------------(2)

To find angle C,

cosC = 72 + 62 - 52

2 x 7 x 5

= 49 + 36 – 25

84

cosC = 0.7143

C = cos-1 0.7143

∴C = 44.40 ---------------- (3)

***Check:*** From Eqn (1), (2) and (3).

A + B + C = 78.50 + 57.10 + 44.40

= 1800

**EVALUATION**

Using cosine rule, calculate the three angles of the following triangles whose sides are given below. Approximate all your answer to the nearest degree.

(1) Δ XYZ, x = 10m, y = 16m and

z = 13m.

(2) Δ PQR, p = 25km, q = 30km, and

r = 8km.

(3) ΔABC, a = 5.7cm, b = 3.5cm and

c = 4.3cm.

**PERIOD 4:**

**GENERAL PROBLEM SOLVING USING SINE AND COSINE RULE.**

A combination of sine and cosine rule can be used to solve a given problem, as we shall see subsequently.

***Example 8:***

*Find the value of the following from the diagram below (i) x (ii) θ (iii) ⎮BD⎮.*

C

13cm

D 430

xcm 7cm

θ 350 1250

A B

***Solution:***

(i) ***Using sine rule***

a = b

sinA sinB

7 = x

sin350 sin1250

X = 7 sin 1250

sin 350

X = 7 sin 550

sin 350

x = 9.99cm

∴ x ≈ 10cm

(ii) ***Using sine rule***

10 = 13

sin430 sinθ

10sinθ = 13sin430

sinθ = 13sin430

10

sinθ = 0.8866

θ = sin-1 0.8866

θ = 620

(iii) To find /BD/

D 13cm C

7cm

B

BCD = BCA + ACD ----------------- (1)

BCA = 1800 – (1250 + 350) *(sum of Δs in ΔABC)*

= 1800 - 1600

= 200

ACD = 180 – (430 + θ0)

= 180 – (430 + 620)

= 180 - 1050

= 750

From (1)

BCD = 200 + 750

= 950

Using cosine rule to find /BD/

/BD/2 = b2 + d2 – 2bdcosC

= 132 + 72 – 2 x 13 x 7cos950

= 169 + 49 – 182[-cos180–95]

= 218 – 182 [-cos 85]

= 218 + 182 x 0.0872

= 218 + 15.87

/BD/2 = 233.87

/BD/ = √233.87

/BD/ = 15.29cm

/BD/ = 15.3cm *(1. d.p)*

***Example 9:***

*Find the unknown sides and angles of a triangle ABC given that C = 690 , a = 9cm and b = 6cm. Give answer to 3 significant figure.*

***Solution:***

A

6cm

690

B 9cm C

***Using cosine rule***

c2 = a2 + b2 – 2abcos C

= 81 + 36 – 108 cos690

= 117 – 108 x 0.3584

= 118 – 38.71

= 79.29

C = √79.29

C = 8.90cm

To get angle B, we shall use sine rule

b = c

sinB sinC

6 = 8.9

sinB sin690

6sin690 = 8.9sinB

sinB = 6sin690

8.9

sinB = 0.6294

B = sin-1 0.6294

∴ B = 390

To get angle A,

A + B + C = 1800 *[sum of Ls in a Δ]*

A + 390 + 690 = 1800

A = 1800 - 1080

∴ A = 720

**EVALUATION**

(1) The figure below is a trapezium

ABCD, in which /AB/ is parallel to

/DC/, and the lengths of the sides are

as shown below.

D 7cm C

1080

5cm

6cm

A 9cm B

Calculate the value of the following

(i) /AC/ (ii) ABC

(2) R

2.5cm

S

3cm 8.3cm

P

6.4cm

Q

The figure above is a triangle PQR with the dimension as shown above. Calculate the following (i) RPQ (ii) /QS/

(3) In ΔPQR p:q:r = √3:1:1. Calculate the

ratio P:Q:R in its simplest form.

***(WAEC).***

(4) Calculate the angles of the triangles

whose sides are in the ratio 4:5:3.

(5) Given a triangle PQR, in which

/PQ/ = 13cm, /QR/ = 9cm,

/PR/ = 7cm and QR is produced to S

so that /RS/ = 6cm. Calculate the

following. (i) cos PRS (ii) /PS/

(6) Find the value of the following from

the diagram below.

(i) x

(ii) DAB

D 8cm C

7.3cm xcm 6cm

A 9.5cm B

**PERIOD 5**

**WEEK 9:**

**BEARINGS**

This is a system of measuring the location of points on the earth’s surface in relation to another using the four cardinal points of the earth. i.e. the North, South, East and West.

There are two major ways of measuring the bearings of points. They are

(i) The three-digit bearing (True bearing).

(ii) The points of compass bearing.

**The Three-digit bearing or True bearing**

This type of bearing is normally expressed using three digits as the name implies e.g. 0030, 0070, 0250, 0670, 1250, 2180 e.t.c.

The bearing is normally read from the North Pole in a clockwise direction until the desired point is reached.

###### **Example 1:**

*The bearing of B from A is 0750, what is the bearing of A from B?*

***Solution:*** N

B 900

750 900

N 0750

A

The bearing of A from B is

900 + 900 + 750 = 2550

*(This is read from the North Pole at point B)*

###### **Example 2:**

*The bearing of Y from X is 2400, what is the bearing of X from Y?*

***Solution:***

N

x 1800

600

N 600

Y

The bearing of X from Y is 0600

*(This is read from the North Pole at point Y)*

***Example 3***:

*The bearing of Q from P is 1880, what is the bearing of P from Q?*

***Solution:*** N

P 1800

80

N

Q 80

The bearing of P from Q is 0080

*(This is read from the North Pole at Q)*

# The Points of Compass Bearing

This type of bearing is usually read either from the North or South to any of the directions specified, East or West. It is usually started with the letters N or S denoting North or South and it is normally ended with the letters E or W denoting East or West i.e. Nθ0W, Nθ0E, Sθ0W, Sθ0E where θ lie between 0 and 900 (00 <θ< 900).

The first letter N or S as the case may be, signifies the point we are reading from and the last letters E or W signifies the direction we are reading to.

***e.g.***

N650E ⇒ We are reading from the

North 650 towards the East.

S300W ⇒ We are reading from the

South 300 towards the West.

S170E ⇒ We are reading from the

South 170 towards the East.

We shall reframe the three examples under the three-digit bearing using point of compass bearing specifications.

***Example 4:****The bearing of B from A is N750E, what is the bearing of A from B?*

***Solution:***

N B

W E

750

S

N

A 750

W E

S

The bearing of A from B is S750W.

***Example 5:***

*The bearing of Q from P is S80W, what is the bearing of P from Q?*

N P

W E

80S

Q N 80

W E

S

The bearing of P from Q is N80E.

***Example 6:***

*The bearing of Y from X is S600W, what is the bearing of X from Y?*

***Solution:***

N X

W E

600

S N 600 Y

W E

S

The bearing of X from Y is N600E

## NOTE THAT:

*The bearing of a place is said to be due North if it is directly to the North; due South if it is directly down South; due East if it is directly towards the East and due West if it is directly towards the West.*

***Examples:***

\* B is due North of A

**B**

N

W A E

S

\* B is due East of A

N

**A** E **B**

S

\* B is due West of A

N

**A**

W E

**B** S

\* B is due South of A

N

**A**

W E

S

**B**

\* C is North East of B

**C**

N

**B** E

\* B is directly South West of A

N

**A**

W E

S

**B**

# *Example 7:*

Z

N 1500

X 420

Y

1. *From the diagram above, find the value of angle XYZ.*
2. *Suppose YZX = 290, what is the bearing of X from Z?*

***Solution:***

Z

N 1500

X θ3

θ1

θ2420

Y

(i) θ1 +1500 = 1800 *[Ls on st line]*

θ1 = 1800 - 1500

∴ θ1 = 300

θ1 = θ2 *[alternate Ls]*

θ2 = 300

XYZ = θ2 + 420

= 30 + 420

∴ XYZ = 720

(ii) If YZX = 290 , then the bearing of X from Z is 2510

1800 + θ3 +290

but θ3 = 420

1800 + 420 + 290 = 2510

# *Example 8:*

N

B 1330

N 0630

A

C

1. *From the diagram above, find the value of angle ABC.*
2. *Suppose ACB = 250, what is the bearing of A from C?*
3. *What is the bearing of C from A?*

***Solution:***

N

B 1330

θ1 θ2

N0630

A θ5

θ4 θ3 C

1. θ1 = 630 *[alternate Ls]*

θ2 +1330 = 1800 *[Ls on st line]*

θ2 = 1800 – 1330

∴ θ2  = 470

ABC = θ1 + θ2

= 630 + 470

= 1100

1. θ2 = θ3 = 470 *[alternate Ls]*

The bearing of A from C is

= 3600 – (θ3 + θ4)

= 3600 – (470 + 250)

= 3600 – 720

= 2880

(iii) θ4 + ABC + θ5 = 1800 *[sum of Ls in a ∆]*

1100 + 250 + θ5 = 1800

θ5 = 1800 - 1350

∴ θ5 = 450

The bearing of C from A is

= 0630 + θ5

= 0630 + 450

= 1080

**EVALUATION**

**(1)**

1. The bearing of P from Q is 1300, what is the bearing of Q from P?
2. The bearing of Y from X is 3200, what is the bearing of X from Y?
3. The bearing of Sokoto from Lagos is 0380, what is the bearing of Lagos from Sokoto?
4. The bearing of B from A is 2300, what is the bearing of A from B?
5. The bearing of B from A is 0780, what is the bearing of A from B?

(2) 2: Express the answers of the following bearing using the points of compass bearing.

1. The bearing of B from A is 2100, what is the bearing of A from B?
2. The bearing of Y from X is 1050, what is the bearing of X from Y?
3. The bearing of Q from P is 3270, what is the bearing of P from Q?
4. If B is directly north-west of A, what is the bearing of A from B?
5. If Q is directly south-east of P, what is the bearing of P from Q?

(3): Express the answers of the following

bearings using the three-digit bearing specification method.

1. The bearing of Q from P is N370E, what is the bearing of P from Q?
2. The bearing of B from A is N490W, what is the bearing of A from B?
3. The bearing of R from S is S680W, what is the bearing of S from R?
4. The bearing of U from V is S230E, what is the bearing of V from U?

(4) (a) C

N 1320

A

N

0350

B

1. From the diagram above, find the value of ABC.
2. If BCA = 400, what is the bearing of A from C?

(b) X

Z 2230

N

Y 3410

1. Find ZYX
2. If XZY = 850, what is the bearing of X from Z?

**References**

1. Fundamental General Mathematics for Senior Secondary School, By Idode G. O
2. New General Mathematic for Senior Secondary School, Book 1, By Channon , Smith Et al.

**WEEK 10:**

**Subject: Mathematics**

Class: SS 2

**TOPIC: *PRACTICAL PROBLEMS ON BEARINGS***

**Content:**

* Three points movement with distance given
* Three points movement with speed and time given
* Two directions with distance given
* Two directions measured with speed and time given
* **special cases**

**PERIOD 1 and 2:**

**SUB-TOPIC: THREE POINTS MOVEMENT WITH DISTANCE GIVEN**

# *Example 1:*

*A dragonfly flew from point A to point B, 25m away on a bearing of 0670. It then flew from point B to point C 17m away on a bearing of 1430.*

*(a) How far is the dragonfly from the*

*starting point to the nearest metre?*

*(b) What is the bearing of the starting point from the dragonfly?*

***Solution:***

We shall represent the movement of the dragonfly with a diagram.

N

B

1430

θ1 θ2

25m 1040 17m

θ3

C

N 0670

b

**A**

HOW TO FIND THE ANGLE B

θ1 = 670 *[alternate angles]*

θ2 + 1430 = 1800 *[sum of <s on a straight line]* θ2 = 180 – 143

θ2 = 370

B = θ1 + θ2

= 670 + 370

= 1040

(a)

**Using cosine rule**

b2 = a2 + c2 – 2ac cos B

= 172 + 252 – 2x17x25 cos 1040

= 289 + 625 – 850 (-cos 760)

= 914 + 850 x 0.2419

= 914 + 205.6

= 1119.6

b = √1119.6

b = 33.46

∴ b = 33m *(nearest metre)*

∴ The dragonfly is approximately 33m from the starting point.

*(b)* ***Using sine rule***

b = c

sin B sin C

33.46 = 25

sin 104 sin C

33.46 sin C = 25 sin 1040

sin C = 25 sin 104

33.46

sin C = 25 sin 760

33.46

sin C = 0.7249

C = sin-1 0.7249

C = 46.470

The bearing of the starting point from the dragonfly is

But θ3 = θ2 = 370*(alternate <s)*

= 360 – (θ3 + C)

= 360 – (370 + 46.47)

= 3600 – 83.470

= 276.50

≈ 2770

# *Example 2:*

*A ship in an open sea sailed from a point A to another point B, 15km away on a bearing of 3100. It then sailed from the point B to another point C, 23km away on a bearing of 0620.*

1. *How far is the ship from the starting point?*
2. *What is the bearing of the starting point from the ship?*

***Solution:***  *(i)* θ4 C

θ3

23km

0620 θ1

B θ2 680 b

15km 500N

θ5

A 3100

To find the angle B,

θ1 + 620 = 900 *[complimentary angles]*

θ1 = 900 - 620

θ1 = 280

θ5 + 500 = 900 *[complimentary angles]*

θ5 = 900- 500

θ5 = 400

θ2 = θ5 = 400 *(alternate angles)*

B = θ1 + θ2

= 280 + 400

= 680

**Using cosine Rule**

b2 = a2 + c2 – 2ac cos B

= 232 + 152 – 2x23x15 cos 680

= 529 + 225 – 690 x 0.3746

= 754 – 258.474

b2 = 495.526

b = √495.526

b = 22.3km

(ii) **Using sine Rule**

b = c

sin B sin C

22.3 = 15

sin 680 sin C

22.3 sin C = 15 sin 680

sin C = 15 sin 680

22.3

sin C = 0.6237

C = sin-1 0.6237

C = 38.60

The bearing of the starting point from the ship is obtained from 3600 – (θ3 + θ4 + C).

θ1 = θ3 *[alternate <s]*

since θ1 = 280

∴ θ3 = 280

= 3600 – (280 + 900 + 38.60)

= 3600 – 156.60

= 203.40

∴ The bearing of the starting point from

the ship is ≈ 2030

**SUB-TOPIC: THREE POINTS MOVEMENT WITH SPEEED AND TIME GIVEN**

(Under this case, we shall be considering the bearing of ONE OBJECT moving to three different points with no distance given but the SPEED AND TIME OF THE VEHICLE GIVEN)

# *Example 3:*

*A boat sails at 50km/h on a bearing of N520E for 1½ hours and then sails at 60km/h on a bearing of S400E for 2 hours.*

1. *How far is the boat from the starting point?*
2. *What is the bearing of the starting point from the boat?*
3. *What is the bearing of the boat from the starting point?*

***Solution:*** N

1. Q θ1400

75km 920

N 520 θ2 120km

P θ4 q θ3

R

Distance PQ = Speed x Time

= (50 x 1½) km

= (50 x 3/2) km

= 75km

Distance QR = (60 x 2) km

= 120km

*To find angle Q,*

θ1 = 520 *(alternate angles)*

Q = θ1 + 400

Q = 520+ 400

= 920

**Using cosine rule**

q2 = p2 + r2 – 2pr cos Q

= 1202 + 752 – 2x120x75 cos 920

= 14400 + 5625 – 18000 [-cos1800-920]

= 20025 – 18000 (-cos 88)

= 20025 + 18000 x 0.0349

= 20025 + 628.2

= 20653.2

q = √20653.2

∴ q = 143.7km.

*(ii)* ***Using sine rule***

q = r

sinQ sin R

143.7 = 75

sin92 sin R

143.7 sinR = 75 sin 920

sinR = 75 sin 920

143.7

sin R = 75 sin 880

143.7

sin R = 0.5216

R = sin-1 0.5216

R = 31.40

But θ3 = 400 *[alternate <s]*

The bearing of the starting point from the boat is = N(R + θ3)0W

= N( 31.40 + 400)W

= N 71.40 W

≈ N 710 W

*(iii)*

But θ2 + 920 + R = 1800 *[<s in a Δ]*

θ2 + 920 + 31.40 = 1800

θ2 = 1800 – 123.40

∴ θ2 = 56.60

θ4 = 1800 –(520 + θ2)

θ4 = 1800 –(520 + 56.60)

θ4 = 1800 –108.60

θ4 = 71.40

∴ θ4 ≈ 710

The bearing of the boat from the starting point is read from the point P as S710E.

# *Example 4:*

*An aircraft flew from an airport A to another airport B, on a bearing of 0650 at an average speed of 300km/h for 21/3 hrs, It then flew from the airport B to another airport C, on a bearing of 3200 at an average speed of 450km/h for 40min.*

1. *How far is the aircraft from the starting point?*
2. *What is the bearing of the starting point from the aircraft?*
3. *What is the bearing of the aircraft from the starting point?*

***Solution:****(i)* θ3C

θ4

300km

b 750 500

θ2 B

650  θ5 2700

N θ1 700km

A

Distance = Speed x Time

Distance AB = (300 x 21/3) km

= (300 x 7/3) km

= (100 x 7) km

= 700km.

Distance BC = (450 x 40) km

60

= (450 x 2/3) km

= (150 x 2) km

= 300km.

since 60min = 1hr

∴ to get time in hrs

= 40

60

= 2

3 hrs

To find angle B,

θ1 + 650 = 900 *[complimentary angles]*

θ1 = 900 – 650

θ1 = 250

θ1 = θ2 *[alternate angles]*

∴θ2 = 250

B = θ2 + 500

∴B = 250 + 500

∴B = 750

***Using cosine rule***

b2 = a2 + c2 – 2ac cos B

= 3002 + 7002 – 2x300x700 cos 750

= 90000 + 490000 – 420000 x 0.2588

= 580000 – 108696

= 471304

b = √471304

∴b = 686.5km

∴ The aircraft is 686.5km from the starting point.

*(ii)* ***Using sine rule***

b = c

sin B sin C

686.5 = 700

sin 75 sin C

686.5 sin C = 700 sin 75

sin C = 700 sin 75

686.5

sin C = 0.9849

C = sin-1 0.9849

∴ C = 800

The bearing of the starting point from the aircraft is read from point C.

i.e. = θ3 + θ4 + C

θ3= 900

θ4 =500 *( alter. Angles)*

= 900 + 500 + 800

= 2200

*(iii)* A + B + C = 1800 *[sum of Ls in a Δ]*

θ5 +750 + 800 = 1800

θ5 = 1800 – 1550

θ5 = 250

The bearing of the aircraft from the starting point is = 900- (θ5 + θ1)

= 900 – (250 + 250)

= 900 – 500

= 0400

(read from the point A)

**EVALUATION**

(1) C

N

9m A

2170

N 4m

B

3200

From the diagram above, find the following

(i) ABC

(ii) /AC/

(iii)The bearing of A from C.

N

(2) P 1220

21km

N

Q

2000

15km

R

From the diagram above, find the following

(i) PQR

(ii) /PR/

(iii) The bearing of P from R.

(3) A town B is 12km from another town A on a bearing of 0470 and another town C is 8km from town B on a bearing of 1240.

(i) How far is town A from town C?

(ii) What is the bearing of town A from C?

(4) A ship sailing in an open sea moves from a point A on a bearing of 0550 at a speed of 50km/h for 1½ hour to another point B. It then moves on a bearing of 1430 at a speed of 40km/h for 2 hours to another point C.

(i) How far is the ship from the starting point?

(ii) What is the bearing of the ship from the starting point?

**PERIOD 3 and 4:**

**SUB-TOPIC: TWO DIRECTIONS WITH DISTANCE GIVEN**

(Under this case, we shall be considering the bearing of TWO OBJECTS at different locations read from the same point or TWO OBJECT moving from the same point in two different directions AND the DISTANCES covered by the two objects GIVEN)

# *Example 5:*

*Two missiles A and B shot from the same point, Missile A was shot on a bearing of 0580 and at a distance of 10km and missile B was shot on a bearing of 1320 at a distance of 18km.*

*(i) How far apart are the missiles?*

*(ii) What is the bearing of missile A from missile B?* ***(WAEC)***

***Solution:***

*(i)* A

10km

N 0580

P θ1

1320 420 740

p

18km

N

B

To get angle P,

θ1 + 580 = 900 *[complementary angles]*

θ1 = 900 - 580

θ1 = 320

P = θ1 + 420

= 320 + 420

= 740

**OR** P = 1320 - 580

= 740

**Using cosine rule**

P2 = a2 + b2 – 2ab cos P

= 182 + 102 – 2x18x10 cos 740

= 324 + 100 – 360 x 0.2756

= 424 – 99.216

= 324.784

P = √324.784

P = 18.0km

The two missiles are 18km apart.

*(ii)* ***Using sine rule***

To find angle B,

b = p

sin B sin P

10 = 18

sin B sin 740

10 sin 74 = 18 sin B

sin B = 10 sin 740

18

sin B = 0.5340

B = sin-1 0.5340

B = 32.30

To get the bearing of A from B

= 2700 + θ2 + B  *[θ2 = 42 (alternate <s)]*

= 2700 + 420 + 32.30

= 344.30

≈ 3440

***Example 6:***

*Two points B and C are observed from a watch tower at point A. If B is 7km on a bearing of 0630 and the other point C is 12km due south of A.*

*(i) How far apart are the two points?*

*(ii) What is the bearing of B from C?*

1. *What is the bearing of C from B?*

***Solution:*** B

7km θ3

0630

A θ1

θ2  1170

a

12km

C

θ1 + 630 = 900 *[complementary angles]*

θ1 = 900 - 630

∴θ1 = 270

θ2 = 900

A = θ1 + θ2

= 270 + 900

= 1170

(i) **Using cosine rule**

a2 = b2 + c2 – 2bc cos A

= 122 + 72 – 2x12x7 cos 117

= 144 + 49 – 168 [-cos 180 – 117]

= 193 – 168 [-cos 63]

= 193 + 168 x 0.4540

= 193 + 76.27

a2 = 269.27

a = √269.27

a = 16.4km

The two points are 16.4km apart.

*(ii)* ***Using sine rule***

to find angle C

a = c

sin A sin C

16.4 = 7

sin 117 sin C

16.4 sin C = 7 sin 117

sin C = 7 sin 63

16.4

sin C = 0.3803

C = sin-1 0.3803

∴C = 22.40

≈ 220

∴ The bearing B from C is 0220

*(iii)* The bearing of C from B is

=180 + θ3

θ3 = C= 220

*(alternate angles)*

= 180 + 220

= 2020

**SUB-TOPIC: TWO DIRECTIONS MEASURED WITH SPEED AND TIME**

(Under this case, we shall be considering the bearing of TWO OBJECTS moving from the same point in two different directions AND the SPEED AND TIME of the vehicles or object’s journey are GIVEN.

***Example 7:***

*From a point on the edge of the sea, one ship A sailed on a bearing of S500E at an average speed of 60km/h for 4 hours and another ship B sailed on a bearing S300W at an average speed of 35km/h for 2 hours.*

*(i) How far apart are the ships?*

*(ii) What is the bearing of ship A from B?*

***Solution:***

*(i)*  P N

70km 300500

N θ1 800

B 240km

θ2

p

A

Distance = Speed x Time

Distance PA = (60 x 4) km

= 240km

Distance PB = (35 x 2) km

= 70km

**Using cosine rule**

P2 = a2 + b2 – 2ab cos P

= 702 + 2402 – 2x70x240 cos 800

= 4900 + 57600 – 33600 x 0.1736

= 62500 – 5832.96

P2 = 56667.04

P = √56667.04

P = 238.05km

The ships are 238km apart.

*(ii)* ***Using sine rule***

To find angle B,

p = b

sin P sin B

238 = 240

sin80 sinB

sin B = 240 x sin 80

238

sin B = 0.9931

B = sin-1 0.9931

∴ B = 83.30

θ1 + B + θ2 = 1800 *[angles on a straight line]*

300 + 83.3 + θ2 = 1800

but θ1 = 30 *[alternate angles]*

θ2 = 1800 – 113.3

θ2 = 66.70

θ2 ≈ 670

∴ The bearing of ship A from B is S670E

***Example 8:***

*Two aircrafts left airport A at the same time one flew on a bearing of 0550 at an average speed of 350km/h and the other flew on a bearing of 2950 at an average speed of 420km/h.*

*(i) How far apart are the aircrafts after two hours.*

*(ii) What is the bearing of the second plane from the first?*

***Solution:*** N

*(i)* B

θ1

a

C 1200

550 700km

840km 650

A 2950

Distance = Speed x Time

Distance AB = (350 x 2) km

= 700km

Distance AC = (420 x 2) km

= 840km

***Using cosine rule***

a2 = b2 + c2 – 2bc cos A

= 8402 + 7002 – 2x840x700xcos 120

= 705600 + 490000 – 1176000 x [-cos 180 – 120]

= 1195600 – 1176000 [-cos 60]

= 1195600 + 1176000 x 0.5

= 1195600 + 588000

a = √1783600

∴a = 1335.5km

The aircrafts are 1335.5km apart.

*(ii)* ***Using sine rule***

a = b

sinA sinB

1335.5 = 840

sin 120 sin B

1335.5 sin B = 840 sin 120

sin B = 840 sin 60

1335.5

sin B = 0.5447

B = sin-1 0.5447

B = 330

To get the bearing of the second ship from the first

Bearing is 1800 + θ1 + B

= 1800 + 550 + 330

= 2680

since θ1 =55 *(alternate angles)*

**EVALUATION**

(1) Two men P and Q set off from a base camp R prospecting for oil. P move 20km on a bearing 2050 and Q moves 15km on a bearing of 0600.Calculate the

(a) Distance of Q from P

(b) Bearing of Q from P

(Give answers in each case correct to the nearest whole number).

***SSCE, June 1996, No 12 (WAEC).***

(2) Two boats A and B left a port C at the same time along different routes. B traveled a distance of 9km on a bearing of 1350 and A traveled a distance of 5km on a bearing of 0620.

(a) How far apart are the two ships?

(b) What is the bearing of ship B from A?

(3) Two flying boats A and B left port P at the same time, A sailed on a bearing of 1150 at an average speed of 8km/h and B sailed on a bearing of 2410 at an average speed of 6km/h.

(a) How far apart are the flying boats after 1½ hour?

(b) What is the bearing of boat A from boat B?

(4) A man observed two boats P and Q at a sea sailing towards him at the point R. He observes P at a bearing of N430W moving at an average speed of 20km/h and Q is on a bearing of S520W moving at an average speed of 30km/h. If P took 2 hours to get to R and Q took 2½ hours to get to R.

(a) How far apart were the two boats when the man first noticed them?

(b) What was the bearing of P from Q?

# *PERIOD 5:*

**SUB-TOPIC:** SPECIAL CASES

# *Example 9:*

*An aeroplane flew from city G to city H on a bearing of 1500. The distance between G and* *H is 300km. It then flew a distance of 450km to city J on a bearing of 0600. Calculate and correct to a reasonable degree of accuracy.*

*(a) The distance from G to J,*

*(b) How far north of H is J,*

*(c) How far west of H is G.*

***SSCE, Nov 1994, No 4 (WAEC****).*

***Solution:***

J N

G

1500

300

x 450km

300km 900 N

θ1  600

y

H

(a) θ1 = 300 *(alternate angles)*

To get the distance from G to J I.e. h since GHJ = 900 we shall use Pythagoras theorem.

h2 = j2 + g2

= 3002 + 4502

= 90000 + 202500

= 292500

h = √292500

h = 540.8

∴ h = 541km *(nearest km)*

(b)

J

x 450km

600

H

Let x be the distance of J north of H.

cos 600 = x

450

x = 450 cos 60

x = 450 x 0.5

x = 225km

∴ J is 225km north of H.

(c) G

300

300km

H

y

Let y be the distance of G west of H.

sin 300 = y

300

y = 300 sin 300

y = 300 x 0.5

y = 150km

∴ G is 150km west of H.

***Example 10:***

*A girl moves from a point P on a bearing of 0600 to a point Q, 40m away. She then moves from the point Q, on a bearing of 1200 to a point R. The bearing of P from R is 2550. Calculate, correct to three significant figures the distance between P and R.*

***SSCE, Nov 1993, No 2b (WAEC).***

***Solution:*** N

Q

1200

θ1 600 θ2

1200 N

40m 450  θ3 R

150

N 600 2550

q

**P**

θ1 = 60 *(alternate angles at P and Q)*

Q = θ1 + 600

= 600 + 600

= 1200

θ2 = 900 – 600 *[complementary angles]*

θ2 = 30

θ3 = θ2 = 300 *(alternate angles)*

R = θ3 + 150

R = 300 + 150

R = 450

To find the distance between P and R represented by q.

***Using sine rule***

q = r

sin Q sin R

q = 40

sin 1200 sin 450

q = 40 sin 120

sin 450

q = 40 sin 600

sin 450

q = 48.98m

∴ q = 49m

**EVALUATION**

**(1) A man travels from a village X on a bearing of 0600 to a village Y which is 20km away. From Y, he travels to a village Z, on a bearing of 1950. if Z is directly east of X, calculate, correct to three significant figures, the distance of (i) Y from Z**

(ii) Z from X.

***SSCE, June 1995, No 10a (WAEC).***

(2) A surveyor standing at a point X sights a pole Y due east of him and a tower Z of a building on a bearing of 0460. After walking to a point W, a distance of 180m in the south-east direction, he observes the bearing of Z and Y to be 3370 and 0500 respectively.

(a) Calculate, correct to the nearest metre.

(i) /XY/

(ii) /ZW/

(b) if N is on XY such that XZ = ZN, find the bearing of Z from N.

***SSCE, June 1998, No 10 (WAEC).***

(3) An aeroplane flies from a town X on a bearing of N450E to another town Y, a distance of 200km. It then changes course and flies to another town Z on a bearing of S600E. If Z is directly east of X, calculate correct to 3 significant figures.

(a) The distance from X to Z.

(b) the distance from Y to XZ.

***(WAEC).***

**GENERAL EVALUATION**

N

(1) A

2100

N 50km

B 1500

80km

C

(a) In the diagram, A, B and C represent three locations. The bearing of B from A is 2100 and the bearing of C from B is 1500. Given that /BA/ = 50km and /BC/ = 80km, calculate:

(i) The distance between A and C correct to the nearest kilometer

(ii) The bearing of A from C to the nearest degree.

(b) How far east of B is C?

***WASSCE, Nov 1999. No 9 (WAEC).***

(2) T

580 N

1610

0530 B

N 15m

18m

A

N

C

In the diagram, three points A, B and C is on the same horizontal ground. B is 15m from A, on a bearing of 0530. C is 18m from B on a bearing of 1610. A vertical pole with top T is erected at B such that angle ATB = 580. Calculate, correct to three significant figures,

(a) The length of AC;

(b) The bearing of C from A;

(c) The height of the pole BT.

***WASSCE, June 2001, N0 12. (WAEC)***

(3) Two planes left Lagos international airport at the same time. The first traveled on a bearing of 0480 at an average speed of 500km/h for 12/5 hour before landing. The second traveled on a bearing of 3320 at an average speed of 400km/h for ¾ hour before landing at its destination.

(a) How far apart are their destinations?

(b) What is the bearing of the first from the second?

**References**

1. Fundamental General Mathematics for Senior Secondary School, By Idode G. O

2. New General Mathematic for Senior Secondary School, Book 1, By Channon , Smith Et al.