WEEK 1

SUBJECT: MATHEMATICS

CLASS: SS2

TOPIC: LOGICAL REASONING

CONTENT:

- Simple and Compound statements.
- Logical operation and the truth table.
- Validity of argument

SIMPLE AND COMPOUND STATEMENTS

Mathematical logic can be defined as the study of the relationship between certain objects such as numbers, functions, geometric figures etc. Statements are verbal or written declarations or assertions. The fundamental (i.e logical) property of a statement is that it is either true or false but not both. So logical statements are statements that are either reasonably true or false but not both.

Example: The following are logical statements;

1. Nigeria is in Africa
2. The river Niger is in Enugu
3. 2 + 5 = 3
4. 3 < 7

p	Q	p^q
T	T	T
T	F	F
F	F	F

N.B The educator should ask the students to give their examples

Example: The following are not logical statements because they are neither true nor false.

1. What is your name?
2. Oh what a lovely day

3. Take her away
4. Who is he?
5. Mathematics is a simple subject (note that this statements is true or false depending on each individual, so it is not logical)
   N.B educator to ask the students to give their own examples

Compound statements—

When two or more simple statements are combined, we have a compound statement. To do this, we use the words: ‘and’, ‘or’, ‘if … then’, ‘if and only if’, ‘but’. Such words are called connectives.

Conjunction (or ^) of logical reasoning: Any two simple statements p,q can be combined by the word ‘and’ to form a compound (or composite) statement ‘p and q’ called the conjunction of p,q denoted symbolically as p^q.

Example: 1. Let p be “The weather is cold” and q be “it is raining”, then the conjunction of p,q written as p^q is the statement “the weather is cold and it is raining”.

2. The symbol ‘^’ can be used to define the intersection of two sets A and B as follows;

$$A \cap B = \{x: x \in A \wedge x \in B\}$$

The truth table for p^q is given below;

Class Activity:

1. Which of the following is (are) simple statement and non statement
   i. The ground is wet
   ii. It is raining
   iii. Go to the front seat
   iv. Base ball is not a sport
   v. Every triangle has four sides
2. In the following problems, determine if the sentence is a statement. Classify each sentence that is a statement as simple or compound. If compound , give the components
   i. Open the door
   ii. 5 is a prime number
   iii. Do you like mathematics

iv. May you live long!
v. Today is Sunday and tomorrow is Monday
vi. Rebecca is studying in class eleven and she has to offer 5 object
vii. 20 is a prime number and 20 is less than 21
viii. Abuja is a city and it is the capital of Nigeria
ix. The earth revolves around the moon
x. Every rectangle is square

LOGICAL OPERATION AND THE TRUTH TABLE

The word 'not' and the four connectives 'and', 'or', 'if … then', 'if and only if' are called logic operators. They are also referred to as logical constants. The symbols adopted for the logic operators are given below.

Logic Operators	Symbols
'not'	$-$ *or* $\sim$
'and'	$\wedge$
'or'	$\vee$
'if … then'	$\rightarrow$
'if and only if'	$\leftrightarrow$

When the symbols above are applied to propositions p and q, we obtain the representations in the table below:

Logic operation	Representation
'not p'	$\sim p$ or $\bar{p}$
'P and q'	$p \wedge q$
'p or q'	$p \vee q$
'if p then q'	$p \rightarrow q$
'p if and only if q'	$p \leftrightarrow q$

CONDITIONAL STATEMENTS AND INDIRECT PROOFS

Many statements especially in mathematics are of the form “if p then q”, such statements are called conditional statements or implications. The statement ‘if p then q’ means p implies q. The p part is called the antecedent (ante means before) whereas the q part is the consequent

Examples:

1. The student can solve the problem only if he goes through the worked examples thoroughly.

Antecedent: The student can solve the problem

Consequent: He goes through the worked examples thoroughly

2. If Dayo is humble and prayerful then he will meet with God’s favour.
   Antecedent: Dayo is humble and prayerful
   Consequent: He will meet with God’s favour

Class Activity:

Identify the antecedent and the consequent in these implicative statements

(a) If I travel then you must teach my lesson
(b) If you person well in your examinations then you will go on holidays
(c) If London is in Britain then 12 is an even number
(d) If the bus come late then I will take a motorcycle
(e) If a & b are integers then ab is a rational number

Converse statements: The converse of the conditional statement “if p then q” is the conditional statement “if q then p” i.e the converse of $p \to q$ is $q \to p$
Example;
Let p be ‘Obi is a boy’ and q be ‘3 + 3 = 4’ and so $p \to q$ is the statement ‘if Obi is a boy then 3 + 3 = 4’. The converse of the statement ($q \to p$) is the statement ‘if 3 + 3 = 4 then Obi is a boy’
(students should give more examples)
Inverse statements: The inverse of the conditional statement “if p then q” is the conditional statement “if not p then not q”. i.e the inverse of $p \to q$ is $\sim p \to \sim q$

Class Activity:

1. Write down the inverse of each of the following statements
   (a) If Mary is a model then she is beautiful
   (b) If Ibadan is the largest city in the west Africa then it is the largest city in Nigeria
   (c) If the army misbehaves again he will be demoted
2. Write down the converse of each of the following
   (a) If he sets a good, he will get a good fellowship
   (b) If it rains sufficiently then the harvest will be good
   (c) If the triangles are congruent then the ratios of their corresponding lengths are equal

VALIDITY OF ARGUMENT

A logical argument is a relationship between a sequence of statements $X_1, X_2, X_3, ..X_n$ called **premises** and another statement Y called the conclusion. Usually, an argument is denoted by $X_1, X_2, X_3, ..X_n$; Y .One of the major application of logic is the determination of validity (correctness) or otherwise of arguments. An argument is valid if its truth T; if the truth value is false F, it is called a fallacy

Examples

1. Test the validity of the following argument with premises X_1, and X_2 and conclusion Y.

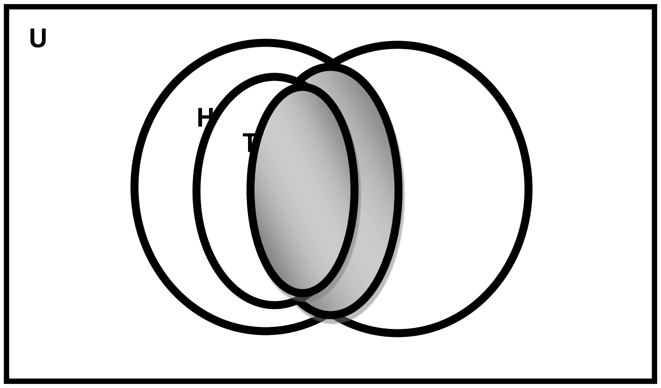
X_1 : All teacher are hardworking.
X_2 : Some young people are teachers
Y : There, some young people are hardworking

Solution

Let

U = { all people}
H = { hardworking people}
N = { young people}
T = { teachers}

The Venn diagrams in the diagram below illustrate this argument



The shaded region of the Venn diagram represent H n N , i.e young people who are hardworking

Since the conclusion follows from the premises, the argument is valid

2. Determine the validity of the following argument
   X_1 : if Bola studies hard he passes his examination
   X_2 : if Tina fails her examination, Bola passes his examination
   X_3 : Bola fails his examination
   Y : therefore, Tina passes her examination
   Solution
   First start by identifying the statement (propositional) variables in argument as follows
   P : Bola studies hard
   Q : Bola passes his examination
   R : Tina passes her examination
   Thus, using the argument form ,X1,X2,X3,.Y i.e (P→q), (r → q) ˜q ; ..r
   Second Construct the relevant truth table

P	Q	R	˜q	˜r	p→q	˜r→q
T	**T**	**T**	**F**	**F**	**T**	**T**
T	**T**	**F**	**F**	**T**	**T**	**T**

T	**F**	**T**	**T**	**F**	**F**	**T**
T	**F**	**F**	**T**	**T**	**F**	**F**
T	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**F**	**F**	**T**	**T**
F	**T**	**F**	**F**	**T**	**T**	**T**
F	**F**	**T**	**T**	**F**	**T**	**T**
F	**F**	**F**	**T**	**T**	**T**	**F**

Class Activity

Determine the validity of the following arguments

1. Bankers are rich.
   Rich people are house owner
   Therefore, bankers are house owners
2. Idle men are never rich
   Wanderers are idle men
   Therefore, a rich man is never wanderer

PRACTICE EXERCISE

1. Determine the validity of the following argument
   i. All reptiles are intelligent animals
   A tortoise is a reptile
   Therefore, a tortoise is an intelligent animal
   ii. No doctor is dirty person
   All friends are clean person
   Therefore, all my friend s are doctors
   iii. Nurses are hospitable people
   My neighbours are hostile to one another
   Therefore, none of my neighbours is a nurse
2. Given the positive intergers x,y,z. prove that if $x<y$ and $y< z$ [Hint: you may use Venn diagram]
3. Prove that if two angles are alternate then the angles are equal

ASSIGNMENT

1. Prove that if a triangle is isosceles, then two of it angles are equal
2. Given two integers m and n. Prove that if m and n are even. Then their products also even
3. Prove that the conditional statement, if $x^2 = 16$, then $x = 4$, is a fallacy
4. Which of the following is the correct interpretation of p v q ?

   A : it will rain tomorrow and the field will be wet

   B: Either it will rain tomorrow or the field will be wet

   C: Either it will rain tomorrow or the field will be wet or it will rain tomorrow and the field will be wet

   D: It will not rain tomorrow but the field will be wet.
5. Determine the validity of the following argument

   X1 : No farmer is lazy

   X2: No non farmer wears gold wrist-watch

   Y: therefore, a lazy person does not wear a gold wrist watch

   KEYWORD: VALID, NEGATION, ARGUMENT, COMPOUND, SIMPLE STATEMENT, PROPOSITION ETC